Turbulence in the Solar Nebula

The Dramatic Role of Magnetic Fields

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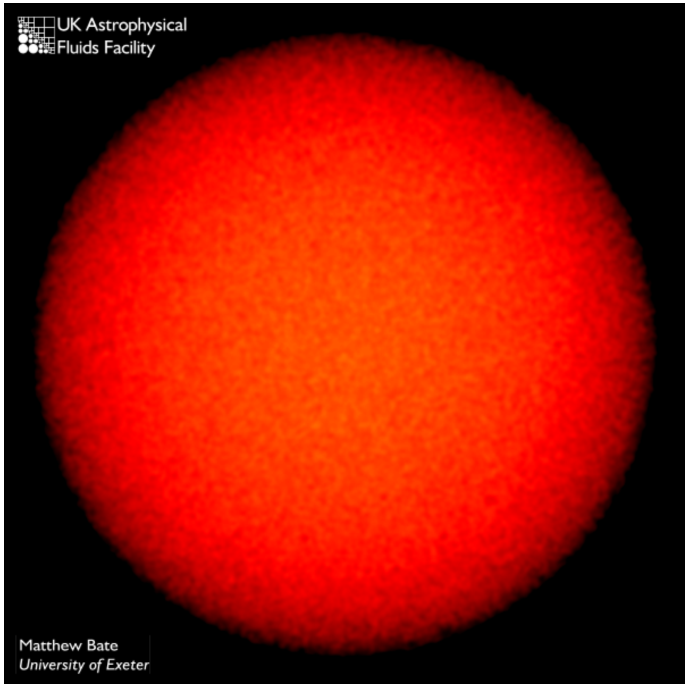
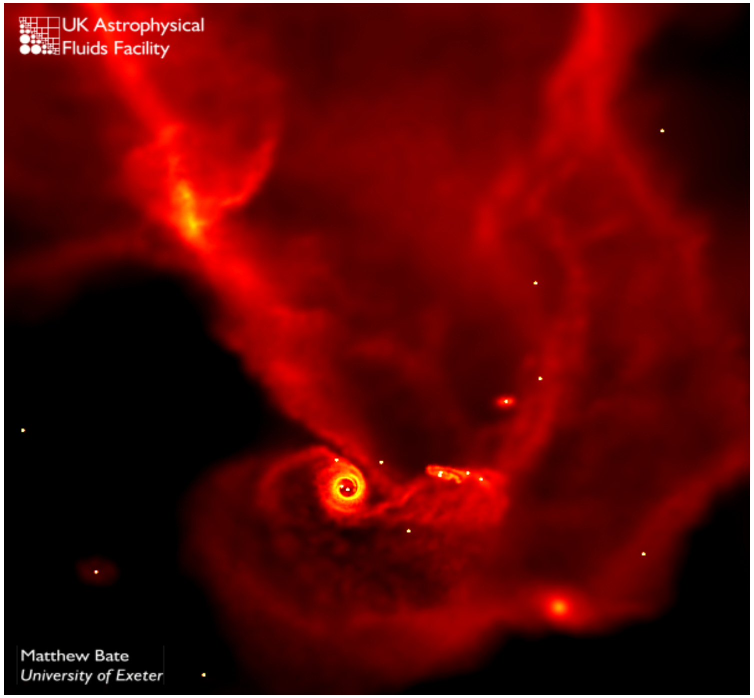
Star Formation - The B3 Simulation (Bate, Bonnell, Bromm 2003)



t=0



t=266 000 yr

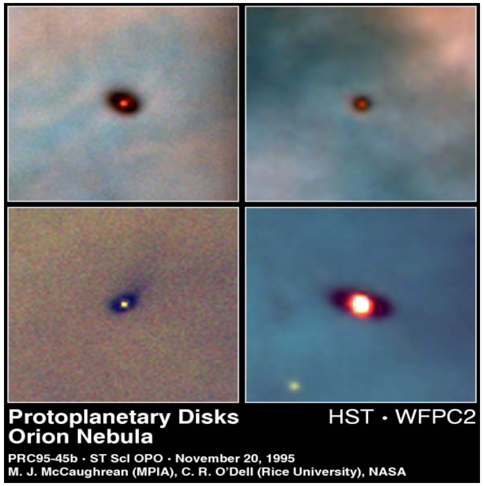
Some stars are seen to be born with lots of surrounding gas.

This gas is bound to the star and referred to as

***circumstellar disk* or *protoplanetary disk*.**

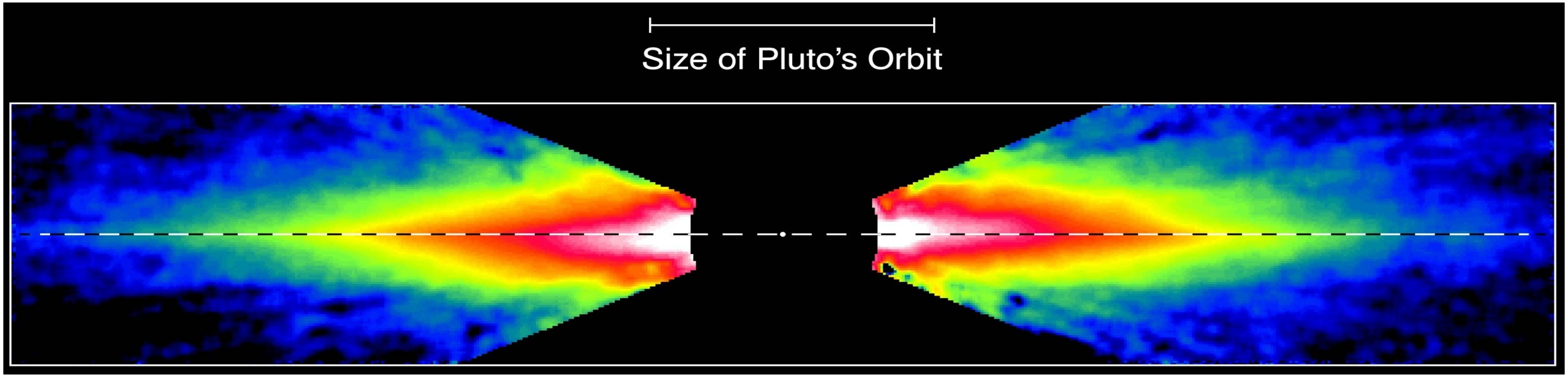
From Solar to "Extra-Solar Nebulae" - Circumstellar Disks

Dust lane
blocks view



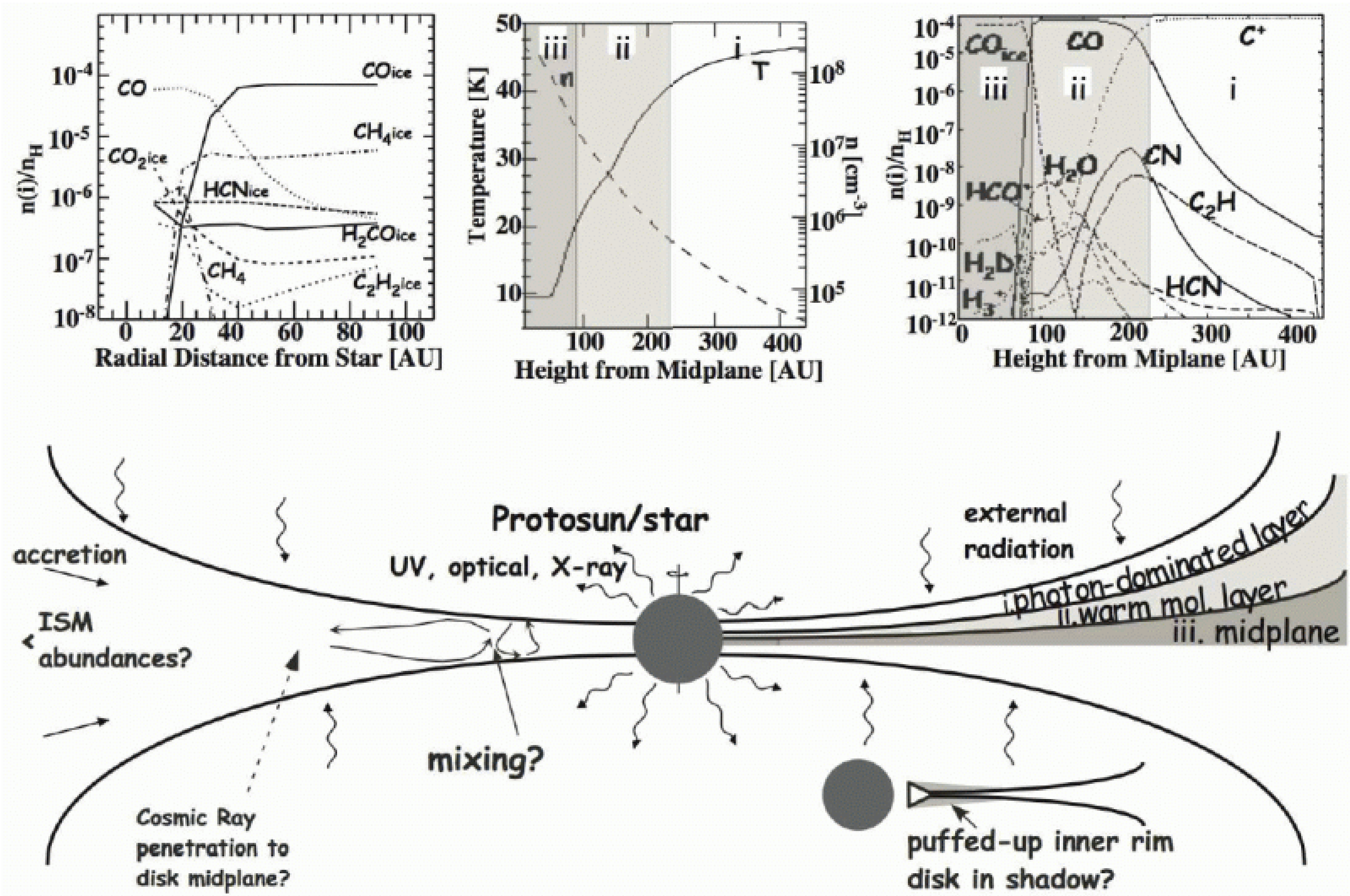
A light background
reveals the disks

Warm dust shines
in infrared



The disk of the star *Beta Pictoris*

Disk Structure



Accretion

300 Kp

Accretion onto a central compact object is believed to power some of the most energetic phenomena in the universe

Black hole accretion
(Lynden-Bell 1969)

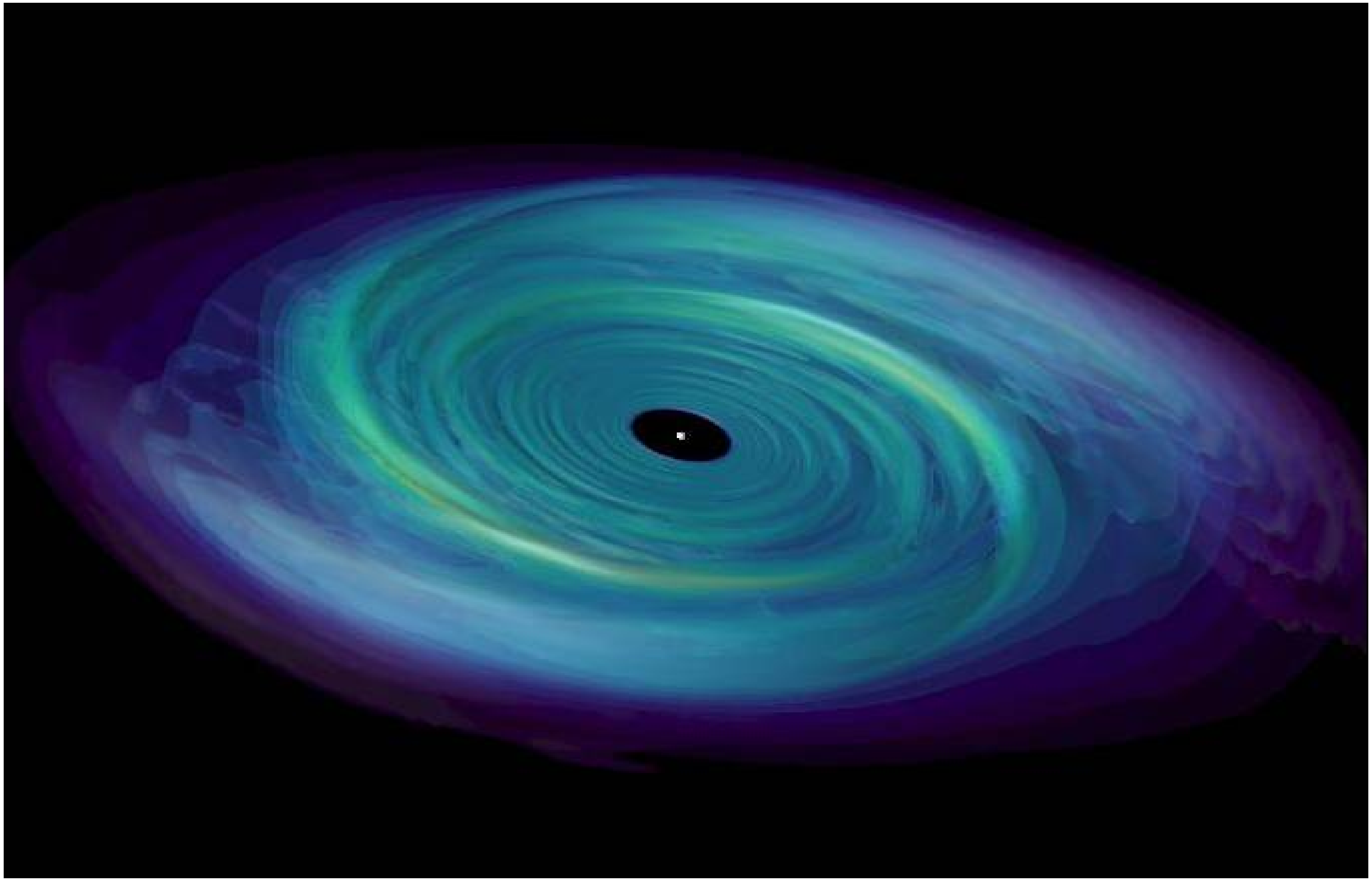
- **Central mass ~10^8-10^{10} M_{sun}**
- **Accretion rate ~1 M_{sun}/yr**
- **Total luminosity ~10^{47} L_{sun}**

"The central problem of nearly 30 years of accretion disk theory has been to understand how they accrete."

Balbus & Hawley (1998)

Angular momentum transport

- **If angular momentum is conserved matter just orbits the central object**
- **Accretion rate is determined by the <u>outward</u> transport of angular momentum**



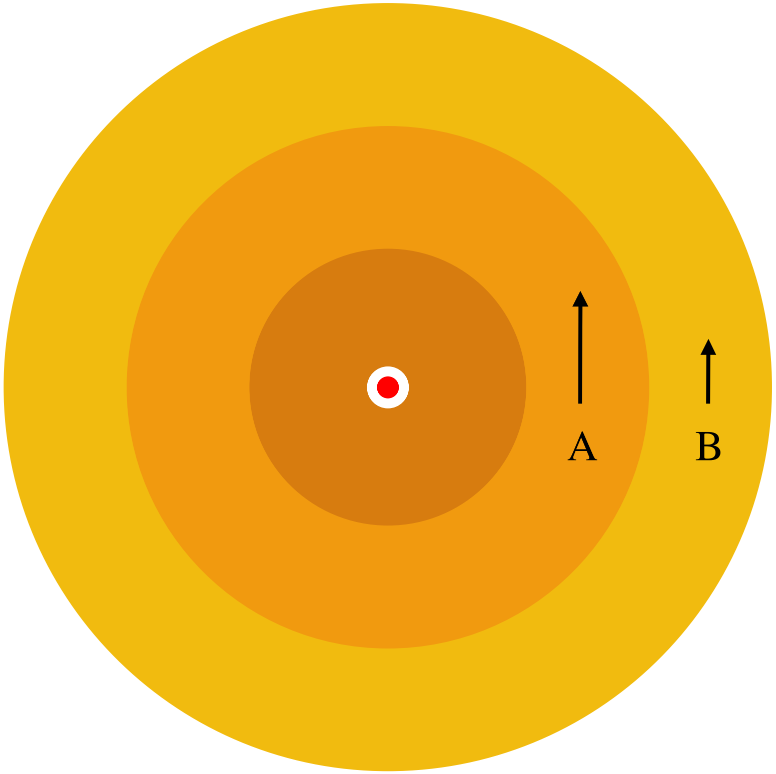
The angular momentum problem

- Angular momentum of 1 M_{sun} in 10 AU disk: 3×10^{53} cm^2/s
- Angular momentum of 1 M_{sun} in 1 R_{sun} star: $\ll 6\times10^{51}$ cm^2/s (=breakup-rotation-speed)
- Original angular momentum of disk = 50x higher than maximum allowed for a star
- Angular momentum is strictly conserved!
- Two possible solutions:
  - Winds (jets) carry angular momentum
  - Very outer disk absorbs all angular momentum by moving outward, while rest moves inward.

Need *friction through viscosity*!

Outward angular momentum transport

Ring A moves faster than ring B. Friction between the two will try to slow down A and speed up B. This means: angular momentum is transferred from A to B.



Specific angular momentum for a Keplerian disk:

$$l = r v_f = r^2 \Omega_K = \sqrt{GM\,r}$$

So if ring A loses angular momentum, but is forced to remain on a Kepler orbit, it must move inward! Ring B moves outward, unless it, too, has friction (with a ring C, which has friction with D, etc.).

The mystery of viscosity

$\nu : cm^2 s^{-1}$

For Molecular Viscosity

$\nu = l^2 / t_{coll} = l V_t$ $\qquad$ $t_{acc} = r^2 / \nu$

For a newly formed disk

$$r = 10^{14} cm ; n = 10^{15} cm^{-3} ; \sigma = 10^{-16} cm^{-2}$$

$$l = 1/(n\sigma) = 10 cm ; V_t = 10^5 cm\, s^{-1}$$

$$t_{acc} = 3 \times 10^{13} yr$$

Observations reveal that disks only live up to $10^7 – 10^8$ yr

MUCH MORE POWERFUL VISCOSITY NEEDED!

Turbulent transport

Shakura & Sunyaev (1973) point that an effective viscosity would arise due to turbulence in the disc.

Euler equation

$$\rho \frac{\partial u_i}{\partial t} = -\rho F_i - \partial_j (p \delta_{ij} + \rho u_i u_j)$$

Turbulence : $q = \overline{q} + \delta q$

Reynolds Stress

$$R_{ij} = \rho \overline{\delta u_i \delta u_j}$$

Reynolds equation

$$\rho \frac{\partial \overline{u}_i}{\partial t} = -\rho \overline{F}_i - \partial_j (\overline{p} \delta_{ij} + \rho \overline{u}_i \overline{u}_j + \rho \overline{\delta u_i \delta u_j})$$

Reynolds stress behaves like a viscous term

$$\rho \overline{\delta u_i \delta u_j} = \nu (\partial_j \overline{u}_i + \partial_i \overline{u}_j)$$

$$\frac{\partial \overline{L_\phi}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{L_\phi} \overline{u}_r) = -\frac{1}{r} \frac{\partial}{\partial r} \left(r^2 \rho \overline{\delta u_\phi \delta u_r} \right)$$

$$\partial_t \overline{L_\phi} + \nabla \cdot (\overline{L_\phi} \boldsymbol{u}) = -\nabla \cdot (r R_{i\phi})$$

Angular Momentum transport is due to the presence of Reynolds stress!

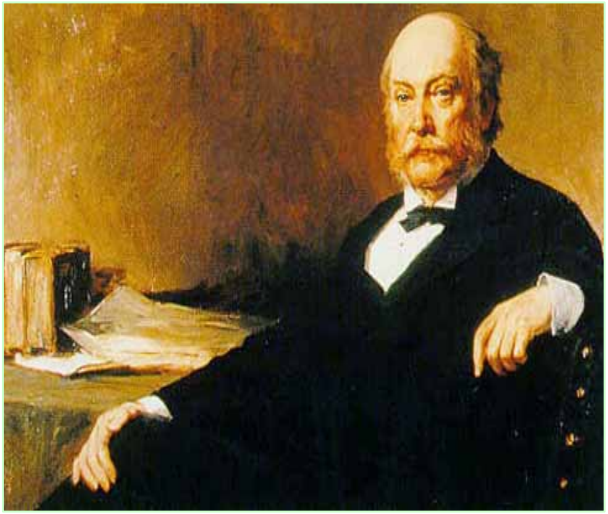
Keplerian disks can't develop hydrodynamical turbulence

Most astrophysical discs are close to Keplerian

- nearly circular
- angular velocity profile $\Omega \propto r^{-3/2}$
  - angular momentum $\left(r^2 \Omega\right)$ increases outwards

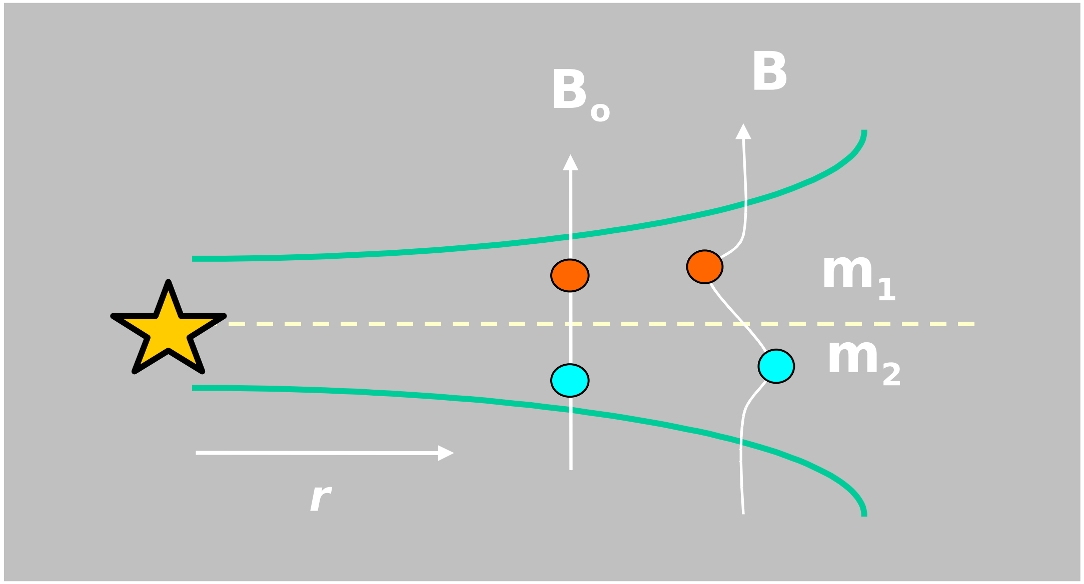
Stable to axis-symmetric disturbances

(Rayleigh criterion)



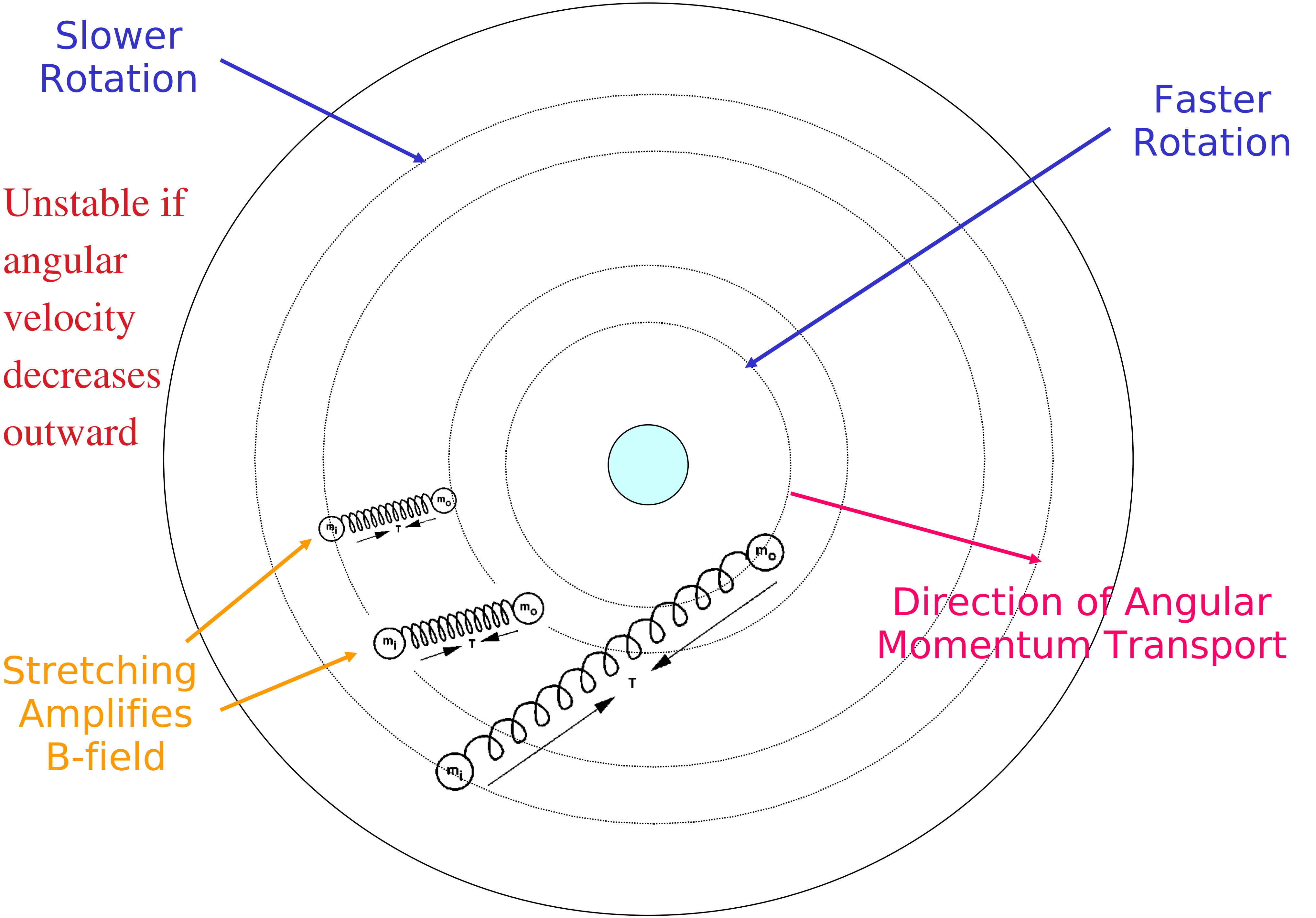
A Magnetic Way to Turbulence...

Stability changes dramatically if disc is even weakly magnetized (Balbus & Hawley 1991)



Tension build-up:

- **tries to restore equilibrium (resists streching)**
- **tries to enforce rigid rotation (resists shearing)**


Slower Rotation
Faster Rotation
Unstable if angular velocity decreases outward
Direction of Angular Momentum Transport
Stretching Amplifies B-field
m_i
m_o
T

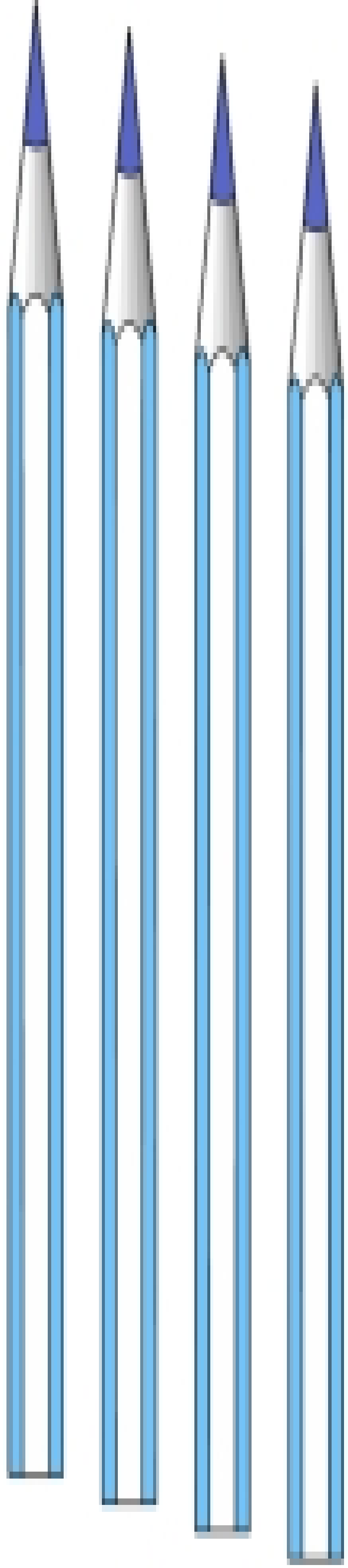
The Rayleigh instability criterion of a negative radial gradient in specific angular momentum is largely irrelevant to gaseous astrophysical disks if magnetic fields are present.

Instead, the combination of differential rotation in the form of a negative angular velocity radial gradient with almost any small seed field will lead to dynamical instability.

Balbus & Hawley 1991

Magneto-Rotational Instability (MRI)

Balbus & Hawley's Accretion Torus simulation



Theoretical Modelling

$$\frac{D \ln \rho}{D t} = -\nabla \cdot \boldsymbol{u}$$

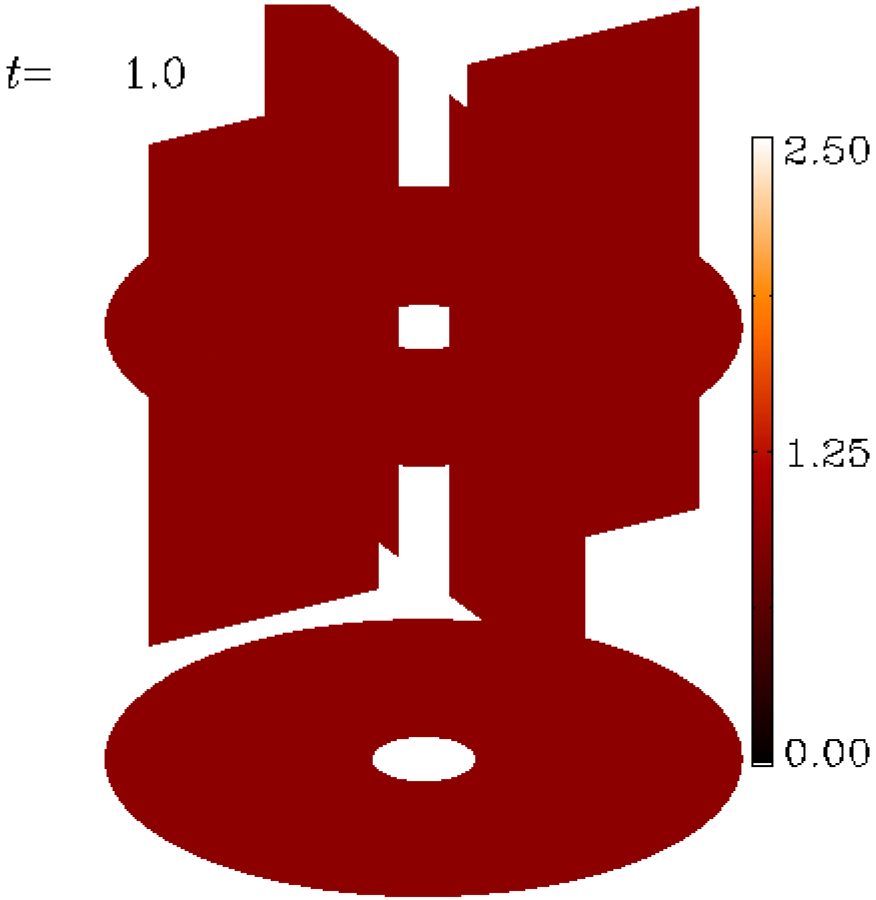
$$\frac{D \boldsymbol{u}}{D t} = -\nabla \Phi_g - \rho^{-1} \nabla p + \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B}$$

The Pencil Code

- High order (3rd order in time, 6th order in space) finite difference MHD code optimized for parallel computations

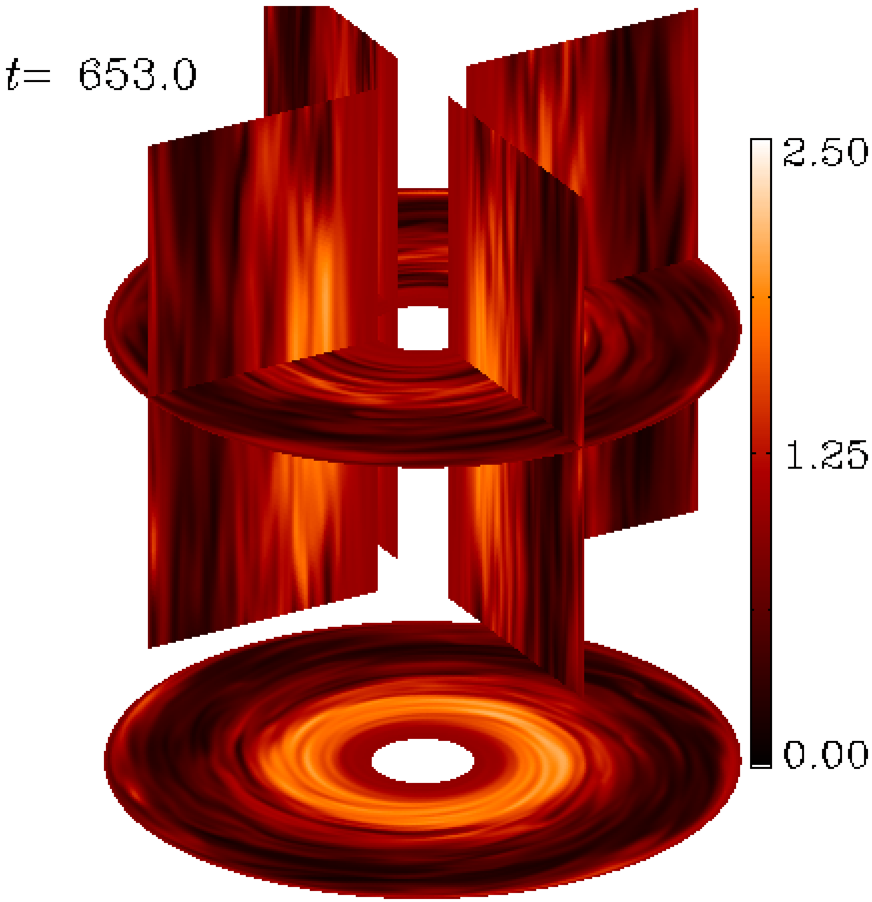
Turbulent Disk Models



Color code: **Density**

Time unit = $(2\mathrm{pi}/T_{\mathrm{Jup}})$ = **1.6 yr**

Density unit = $\mathbf{2x10^{-11}\ g/cm^3}$



[*Density Evolution*]

[*Evolution of magnetic energy*]

Maxwell Stress comes to aid

$$\frac{D \ln \rho}{D t} = -\boldsymbol{\nabla} \cdot \boldsymbol{u}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla \Phi_g - \rho^{-1} \nabla p + \frac{\boldsymbol{J} \times \boldsymbol{B}}{\rho}$$

$$\frac{\partial \boldsymbol{A}}{\partial t} = \boldsymbol{u} \times \boldsymbol{B}$$

Maxwell Stress

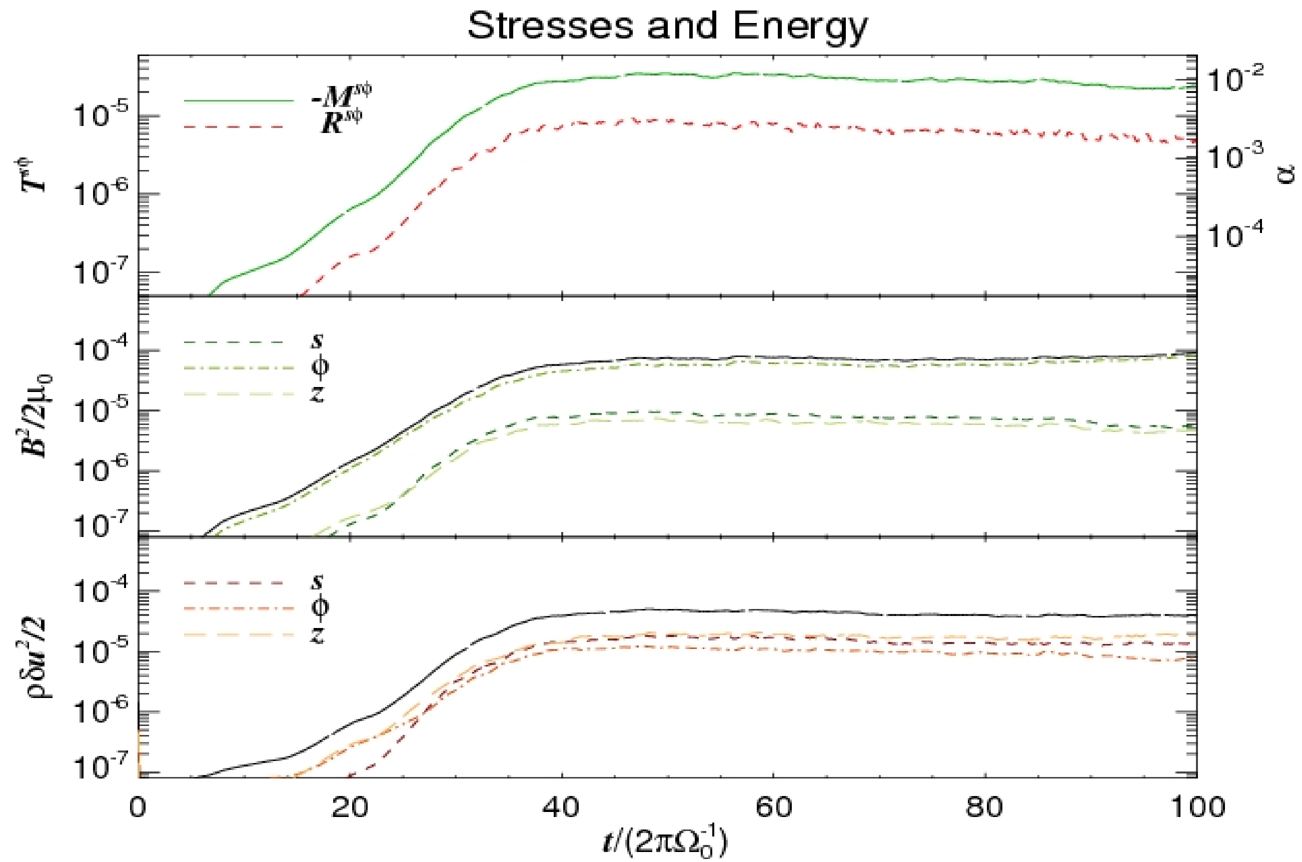
$$M_{ij} = \mu_0^{-1} \overline{\delta B_i \delta B_j}$$

$$\frac{\partial \overline{L_\phi}}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \overline{L_\phi} \overline{u_r}) = -\frac{1}{r} \frac{\partial}{\partial r} \left[r^2 \left(\rho \overline{\delta u_\phi \delta u_r} - \frac{\overline{\delta B_\phi \delta B_r}}{\mu_0} \right) \right]$$

$$\partial_t \overline{L_\phi} + \boldsymbol{\nabla} \cdot (\overline{L_\phi} \overline{\boldsymbol{u}}) = -\boldsymbol{\nabla} \cdot (r (R_{r\phi} - M_{r\phi}))$$

Turbulence stresses transport angular momentum

$$\partial_t \overline{L_\phi} + \nabla \cdot (\overline{L_\phi} \boldsymbol{u}) = -\nabla \cdot (r(R^{r\phi} - M^{r\phi}))$$



Saturation is reached after ~30 orbits

Saturated State of MRI – Brandenburg et al (1995)

Accretion Disk Dynamo – Energy Budget

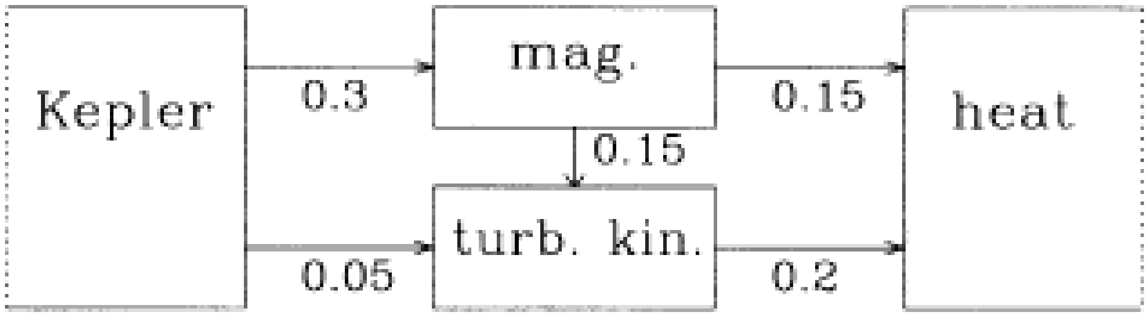


FIG. 6.—Sketch of the energy budget. Energy is tapped from the Keplerian motion and goes into magnetic and kinetic energy, and is finally converted into heat. The numbers give the approximate energy fluxes in units of $\langle \frac{1}{2}\boldsymbol{B}^2\Omega \rangle$.

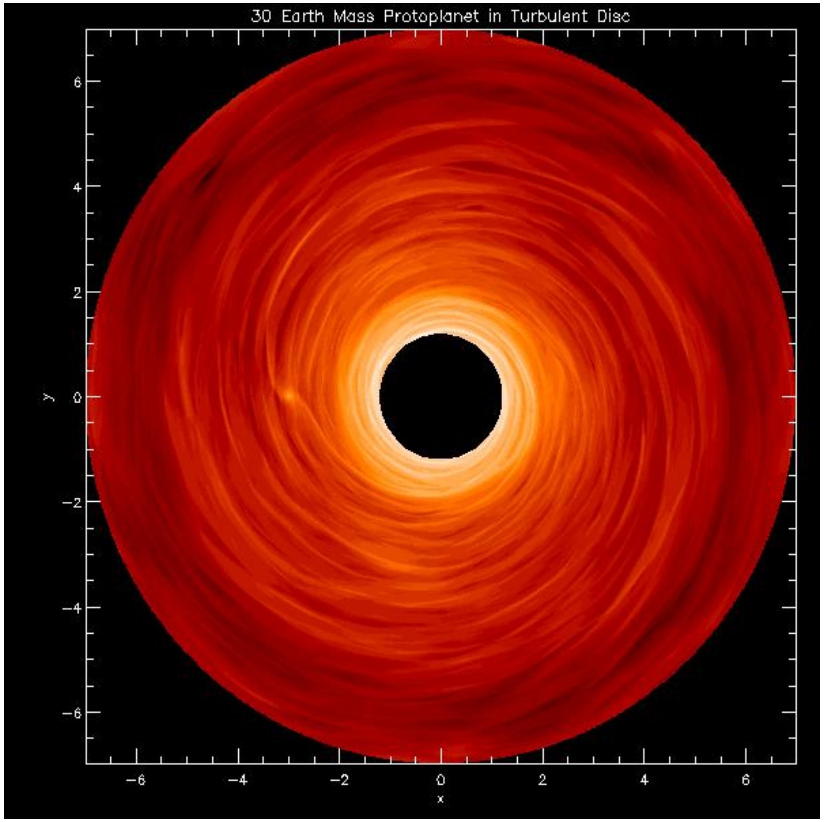
$$\frac{d}{dt}\left(\frac{1}{2\mu_0}B^2\right) = -\frac{3}{2}\Omega_0 \frac{1}{\mu_0} B_x B_y - \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) - \eta\mu_0 J^2$$

$$\frac{d}{dt}\left(\frac{1}{2}\rho u^2\right) = \frac{3}{2}\Omega_0 \rho u_x u_y + \rho\boldsymbol{u}\cdot\boldsymbol{g} - \boldsymbol{u}\cdot\nabla p + \boldsymbol{u}\cdot(\boldsymbol{J}\times\boldsymbol{B}) - 2\nu\rho S^2$$

$$\frac{d}{dt}\rho e = -p\nabla\cdot\boldsymbol{u} + 2\nu\rho S^2 + \eta\mu_0 J^2 + \rho Q$$

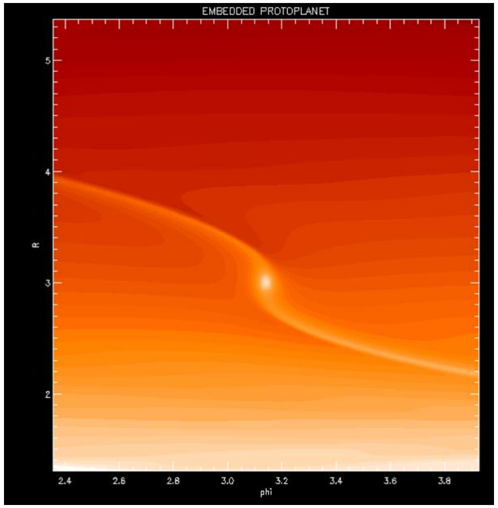
$$\frac{d}{dt}E_{tot} = \frac{3}{2}\Omega_0\left(\rho u_x u_y - \frac{1}{\mu_0}B_x B_y\right) + \rho Q$$

A 30 Earth Mass Planet…

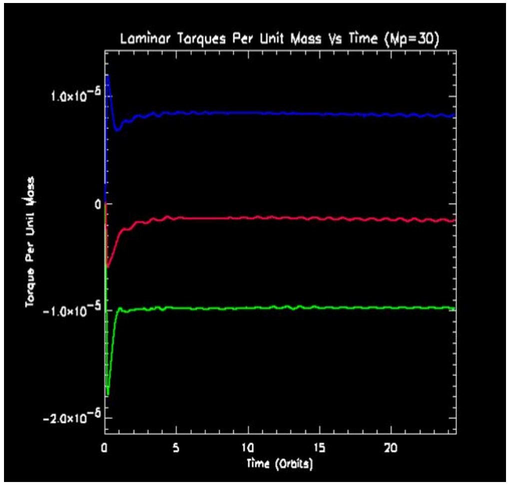


Source: Richard Nelson

Laminar

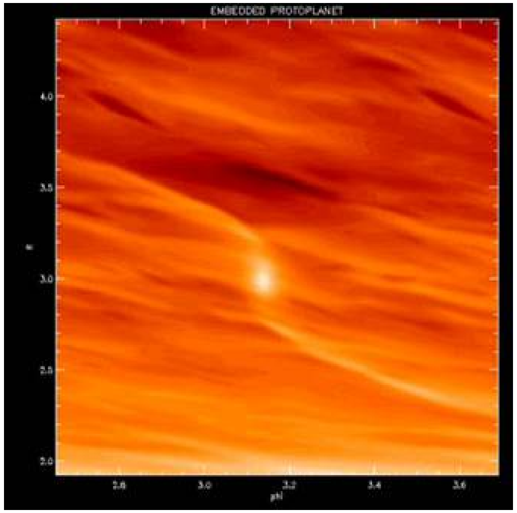


Density

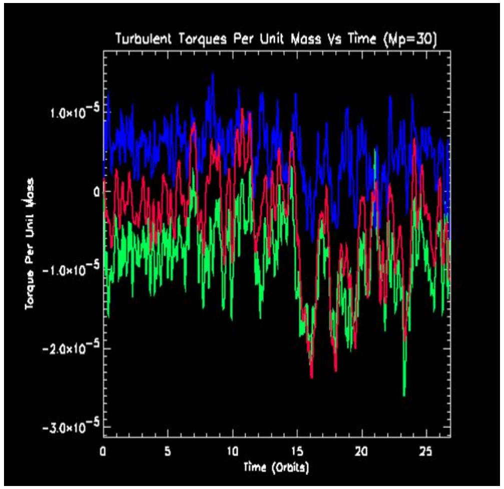


Torques

Turbulent

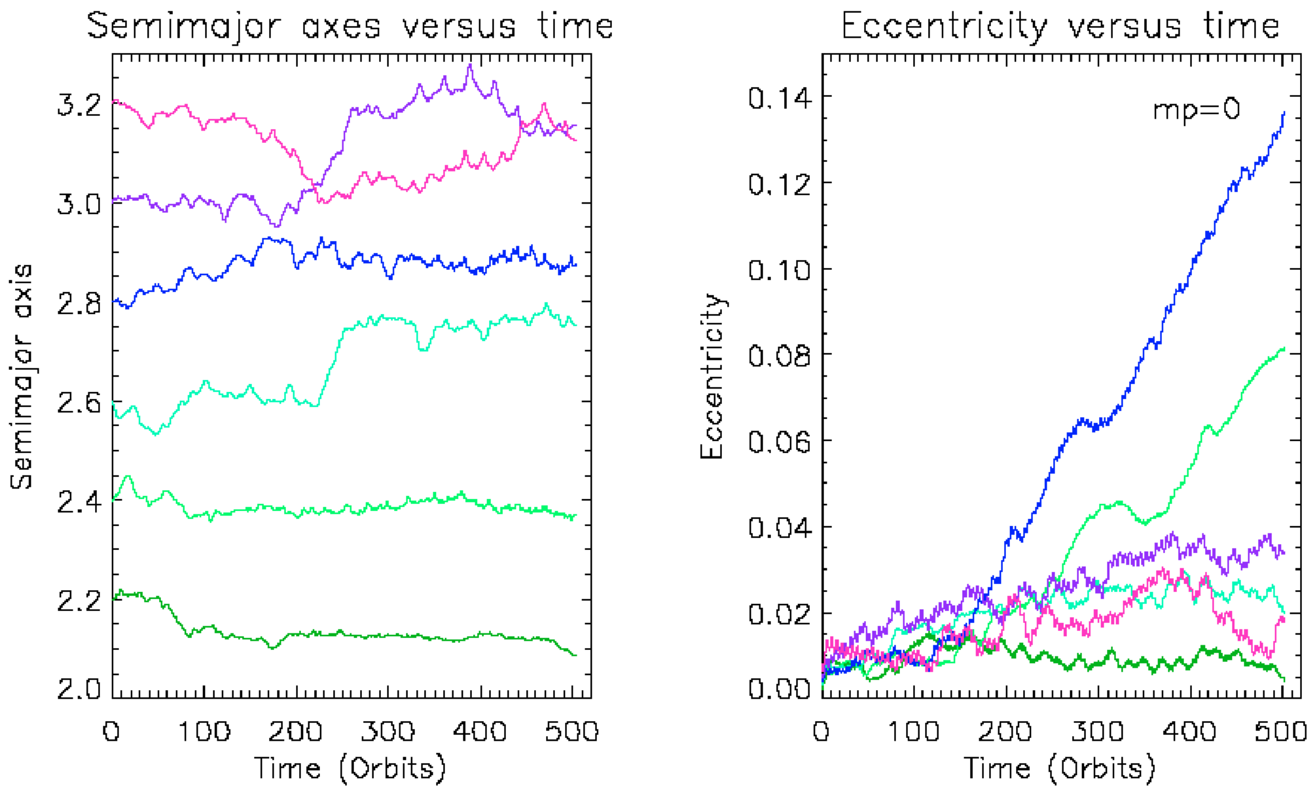


Density



Torques

Planets in Turbulent Disks



*Migration becomes a **stochastic process***

as planets undergo random walk

Source: Richard Nelson

Planet Formation

Planets form in protoplanetary disks from dust grains that collide and stick together (planetesimal hypothesis of Safronov, 1969)

From dust to planetesimals

μm -> m: Contact forces in collisions cause sticking

m -> km: ???

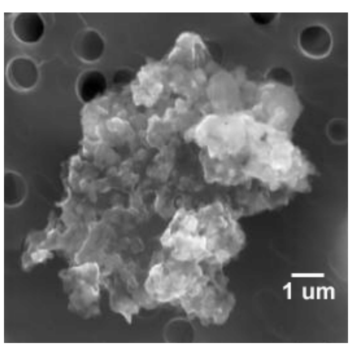
From planetesimals to protoplanets

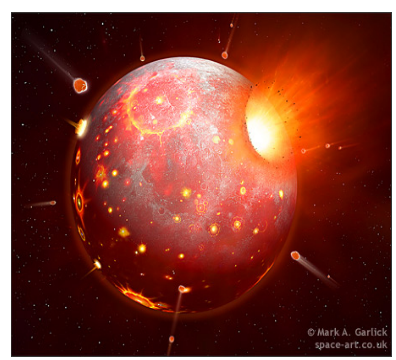
km -> 1000 km: Gravity

From protoplanets to planets

Rocky Planets: Protoplanets collide

Gas Giants: Attract gaseous envelope





From meter to kilometer

Solids are *pressureless* and move under loose influence of gas drag

Solids

$$\frac{d\boldsymbol{w}}{dt} = -\nabla\Phi - \frac{(\boldsymbol{u}-\boldsymbol{w})}{\tau}$$

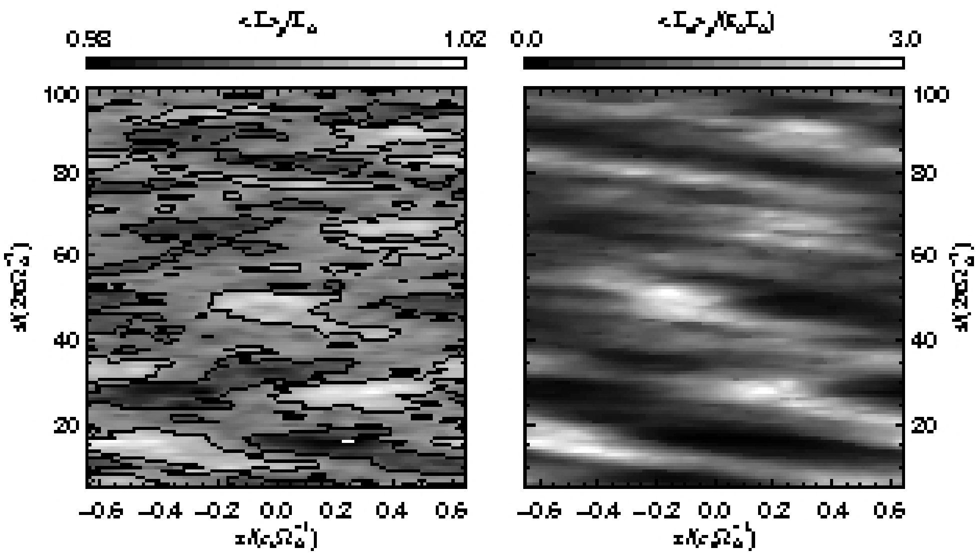
$$\boldsymbol{V} = \boldsymbol{w} - \boldsymbol{u}$$

Gas

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla\Phi - \rho^{-1}\nabla p + \frac{\boldsymbol{J}\times\boldsymbol{B}}{\rho} + \frac{\rho_p}{\rho}\frac{(\boldsymbol{u}-\boldsymbol{w})}{\tau}$$

$$\boxed{\frac{D\boldsymbol{V}}{Dt} \approx \rho^{-1}\nabla p - \frac{\boldsymbol{V}}{\tau}}$$

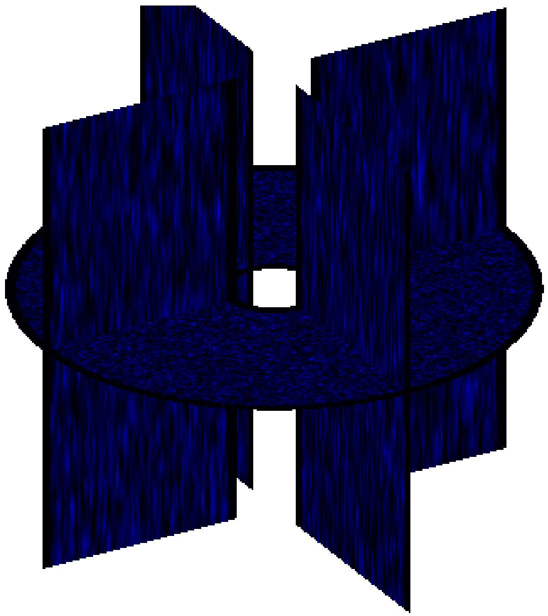
Solids instantaneously move *towards* the pressure gradient

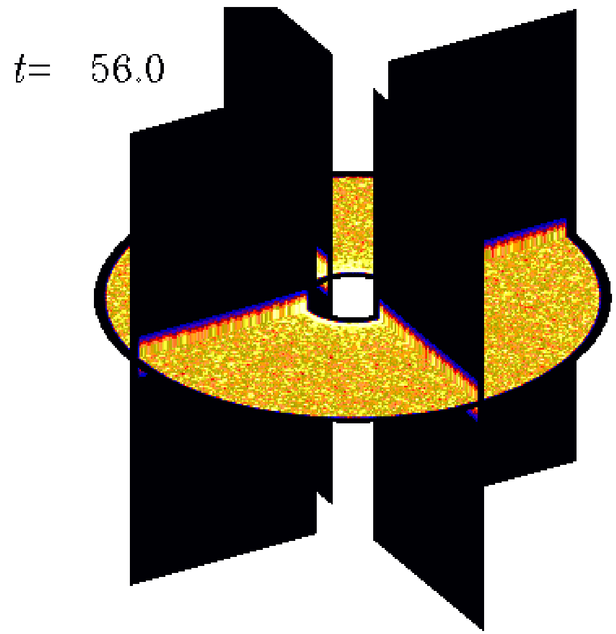


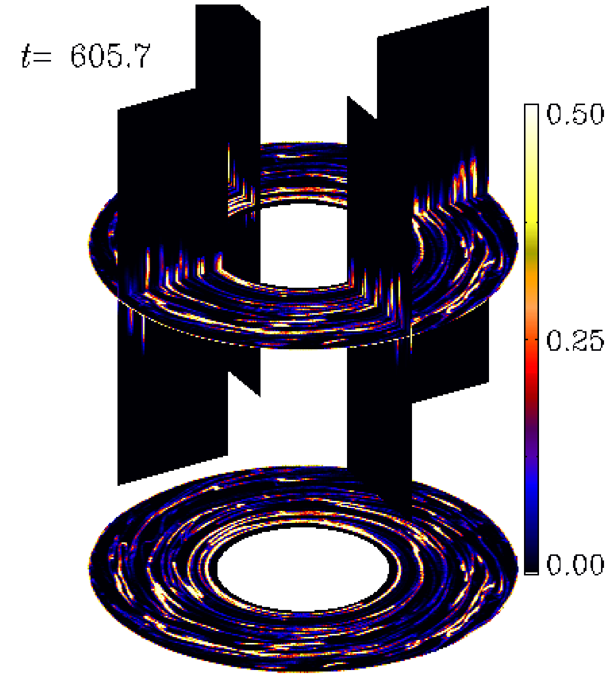
Solids are caught in gas overdensities

Source: Anders Johansen

Trapping Solids







1 million meter sized particles (boulders)

Initial density – interstellar value of dust-to-gas ratio of 0.01

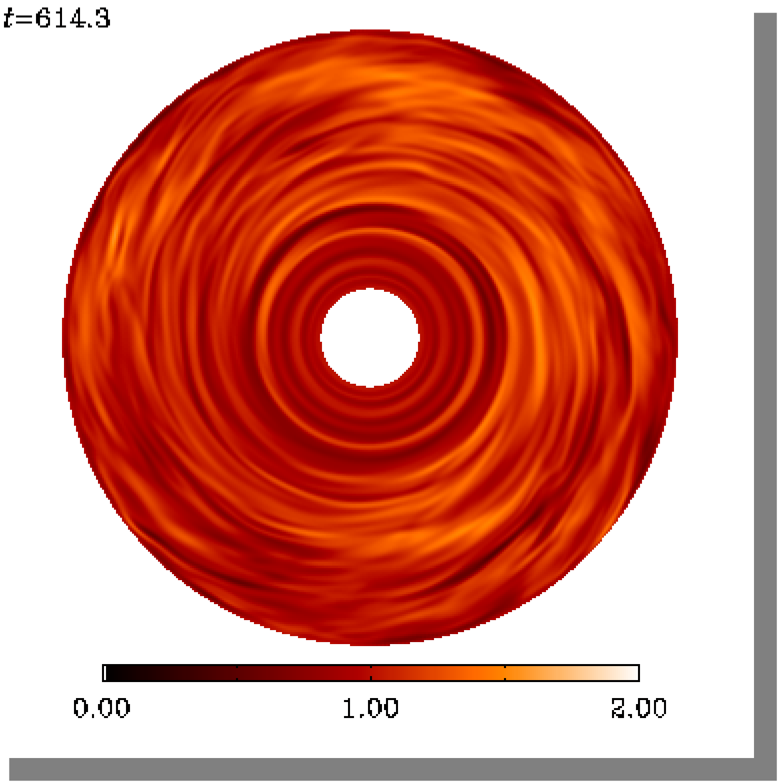
Laminar Model

Homogeneous

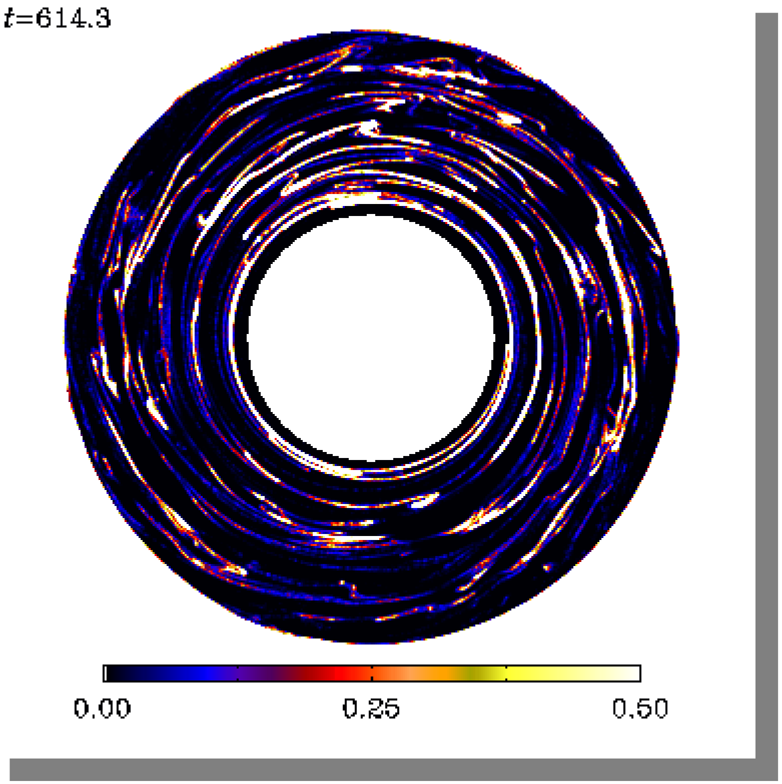
Turbulent Model

Intense Clumping

Turbulent eddies are efficient particle traps



Gas Density

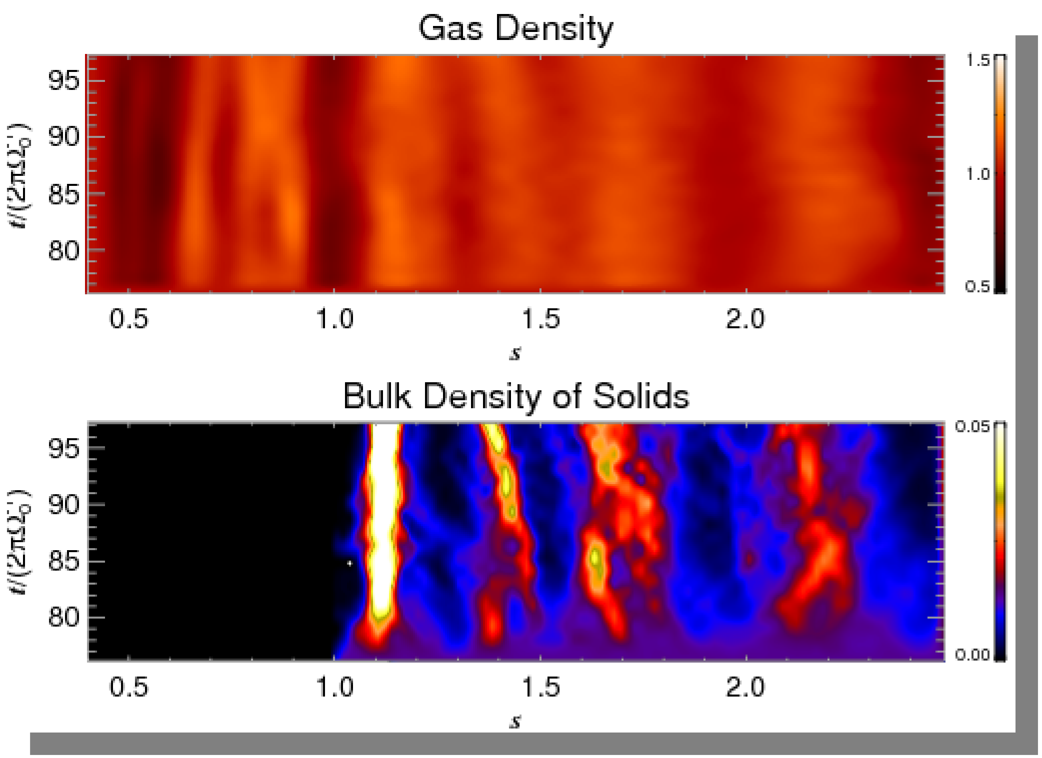


Bulk Density of Solids

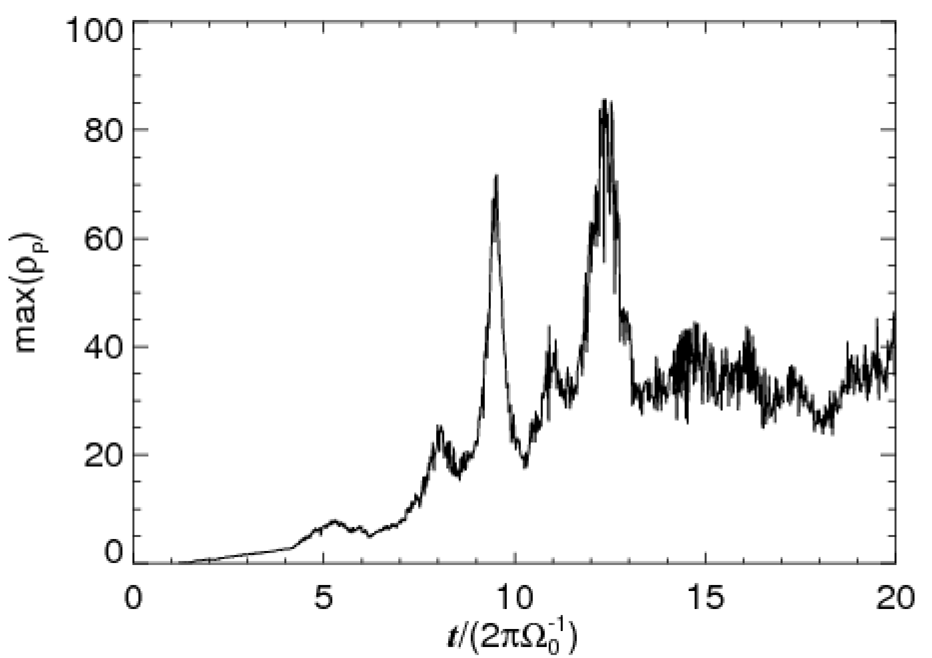
Correlation between pressure maxima and maxima of particle concentration

Planetesimal Formation

Factor 2 in pressure



2 orders of magnitude in particle density!



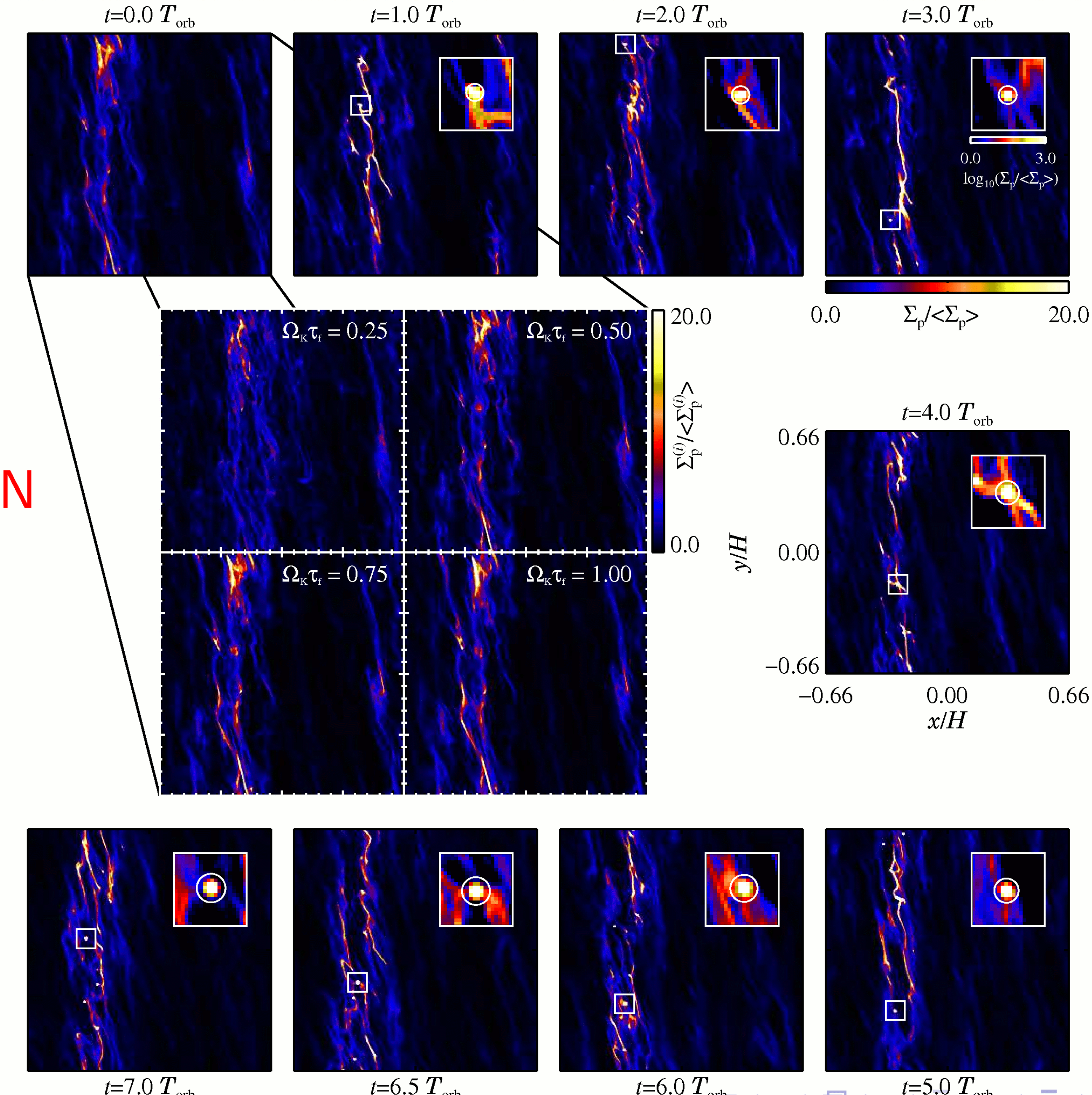
Does collapse occur when considering the self-gravity of the swarm of solids?

Check [***this simulation***] by Anders Johansen!

Clump condensation

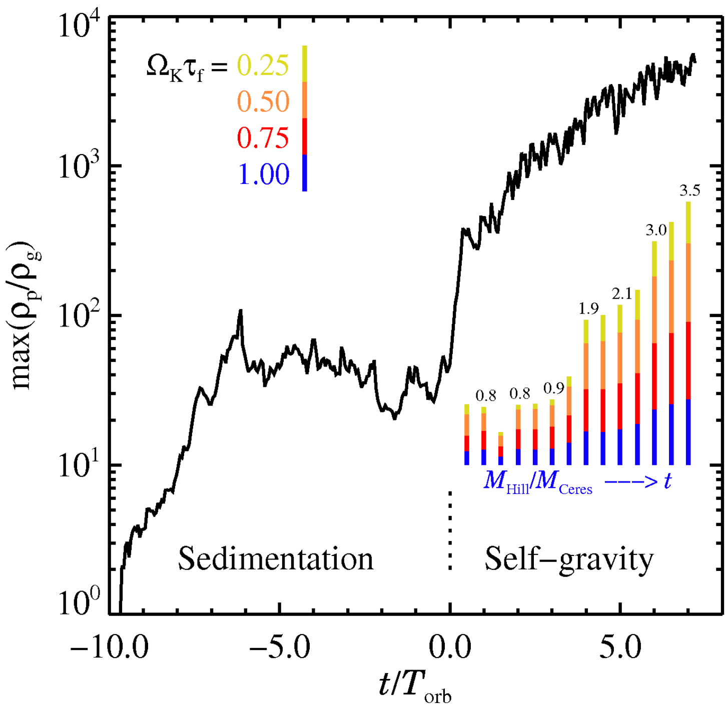
$2 \times$ MMSN

$\epsilon = 0.01$



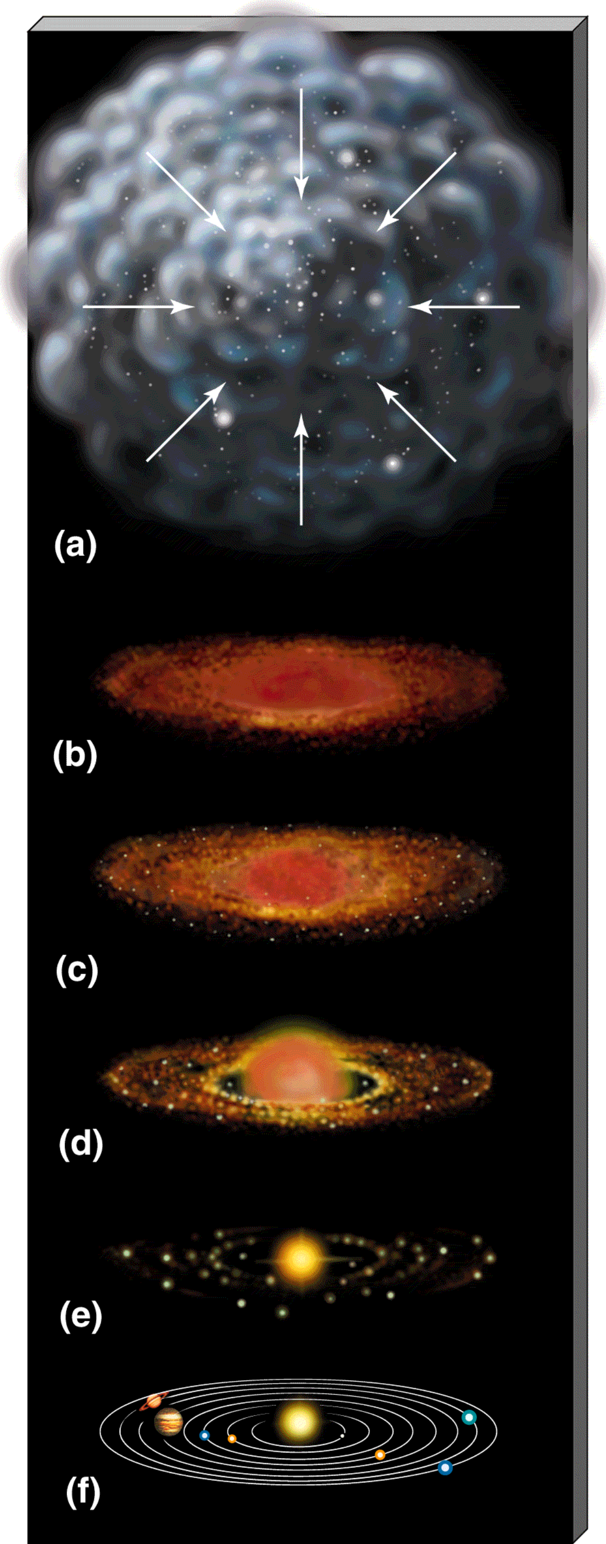
Accretion

- Turbulent concentrations and streaming instability interact constructively and produce overdensities of several 100 in the mid-plane layer
- Gravitationally bound clumps condense out even in discs comparable to minimum mass solar nebula.
- Differential radial drift of different particle sizes does not disrupt the collapse
- Clumps have masses similar to dwarf planets and continue to accrete.



- Growth from boulders to planetesimals does not rely on sticking efficiency.
- Collapse happens much faster than the radial drift time-scale.

Johansen, Oishi, Mac Low, Klahr, Henning, & Youdin (submitted)



Summarizing...

Gravitational collapse of an interstellar cloud

Outward transport of angular momentum through turbulence generated by the MRI

Rocks in the turbulent medium are trapped in transient pressure maxima and undergo collapse into planetesimals

Planetesimals collide and give rise to a planetary system