Regional modeling of electric field using EISCAT3D plasma velocity measurements

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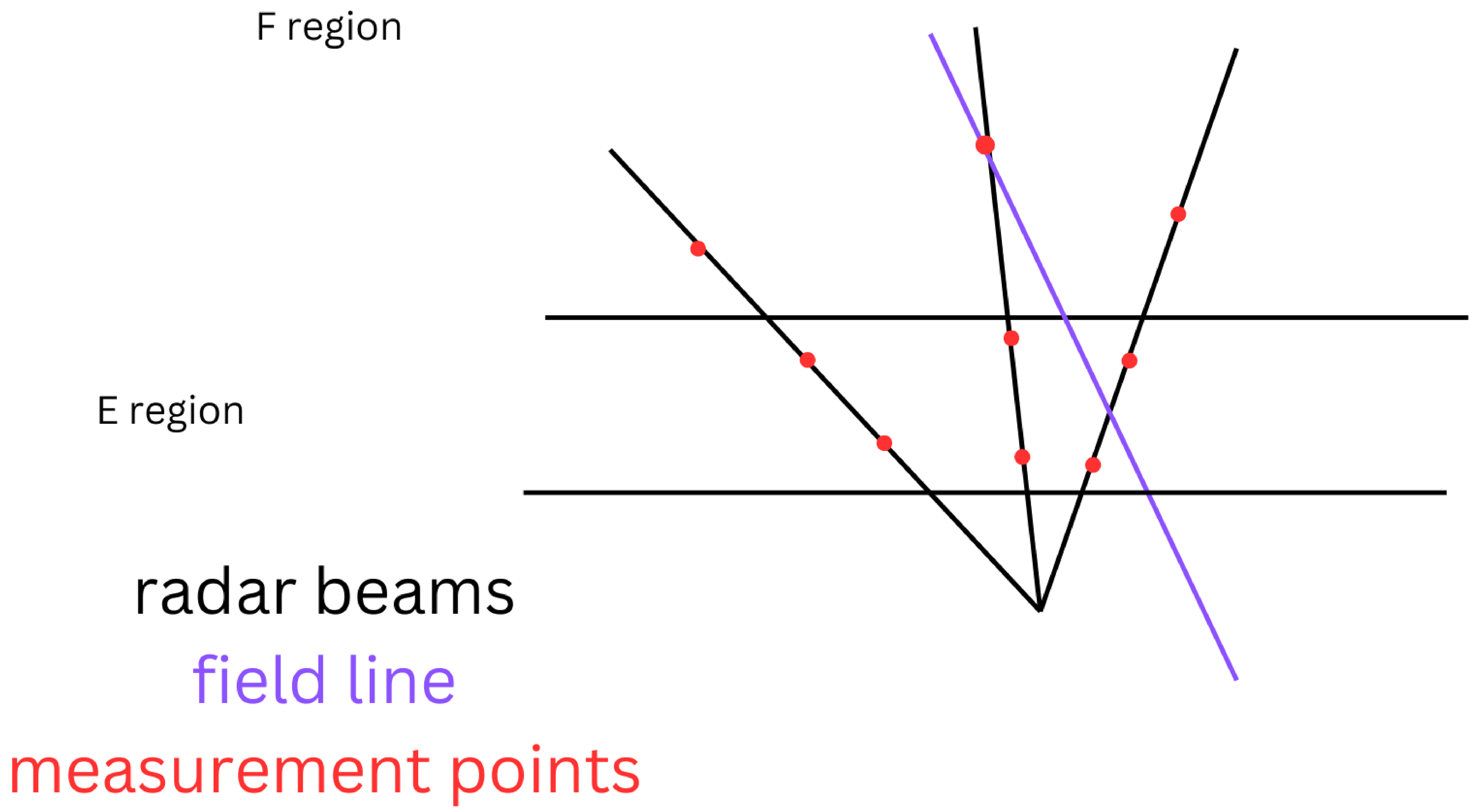
Introduction

- EISCAT3D will provide us the 3D ion velocity $\boldsymbol{V}_i$
- In the F region, the electric field can be estimated as

$$\boldsymbol{E} = \boldsymbol{B} \times \boldsymbol{V}_i$$

- What about E region?
- The F region electric field estimate can be mapped to the E region
- But it can't be mapped to points where E3D make observation



Spherical elementary system (SECS)

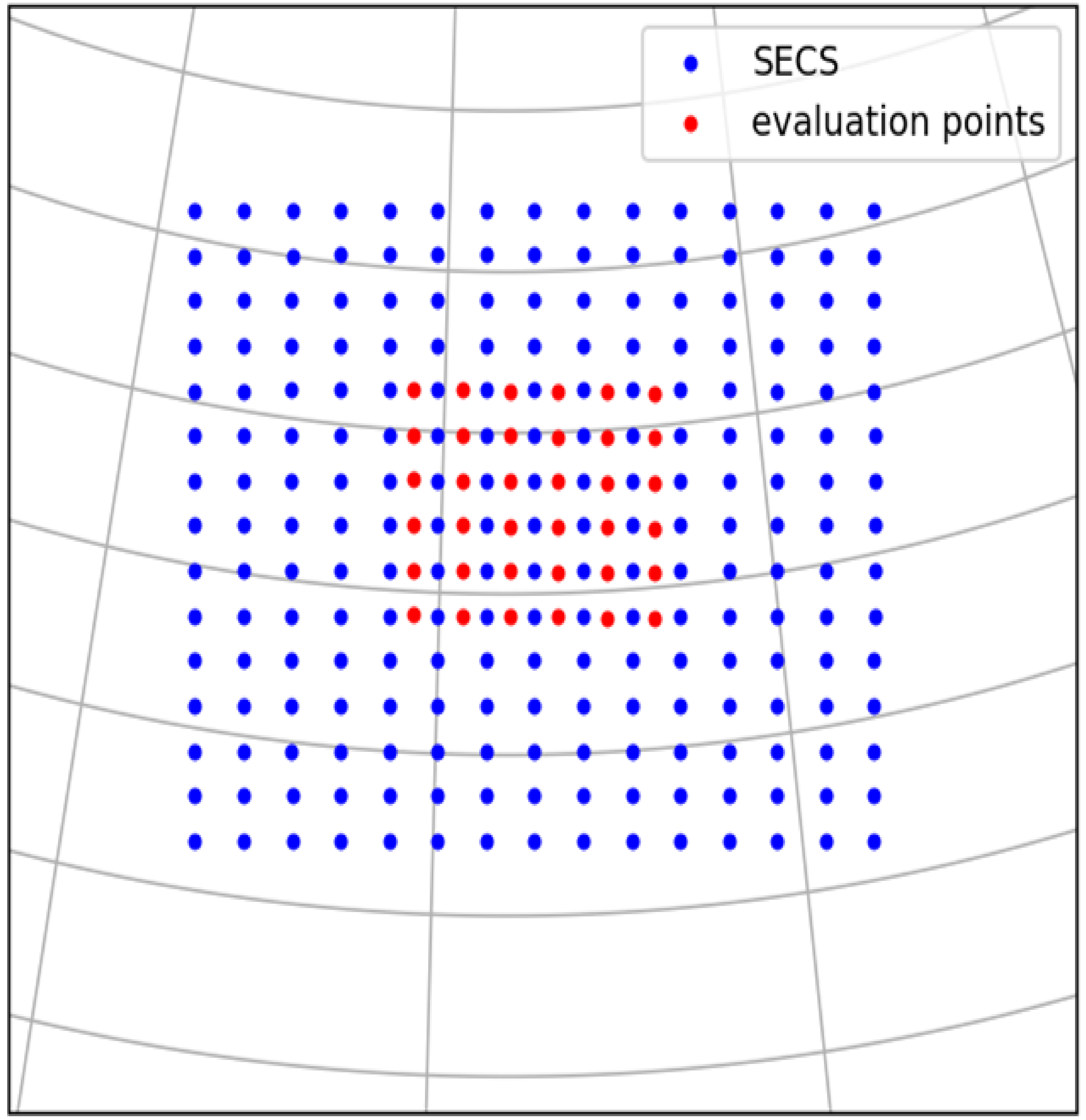
- Any 2-D vector field on a spherical surface can be represented as

$$\begin{bmatrix} \boldsymbol{E}_e \\ \boldsymbol{E}_n \end{bmatrix} = \boldsymbol{GS}$$

- ***G*** is geometry matrix of the SECS system

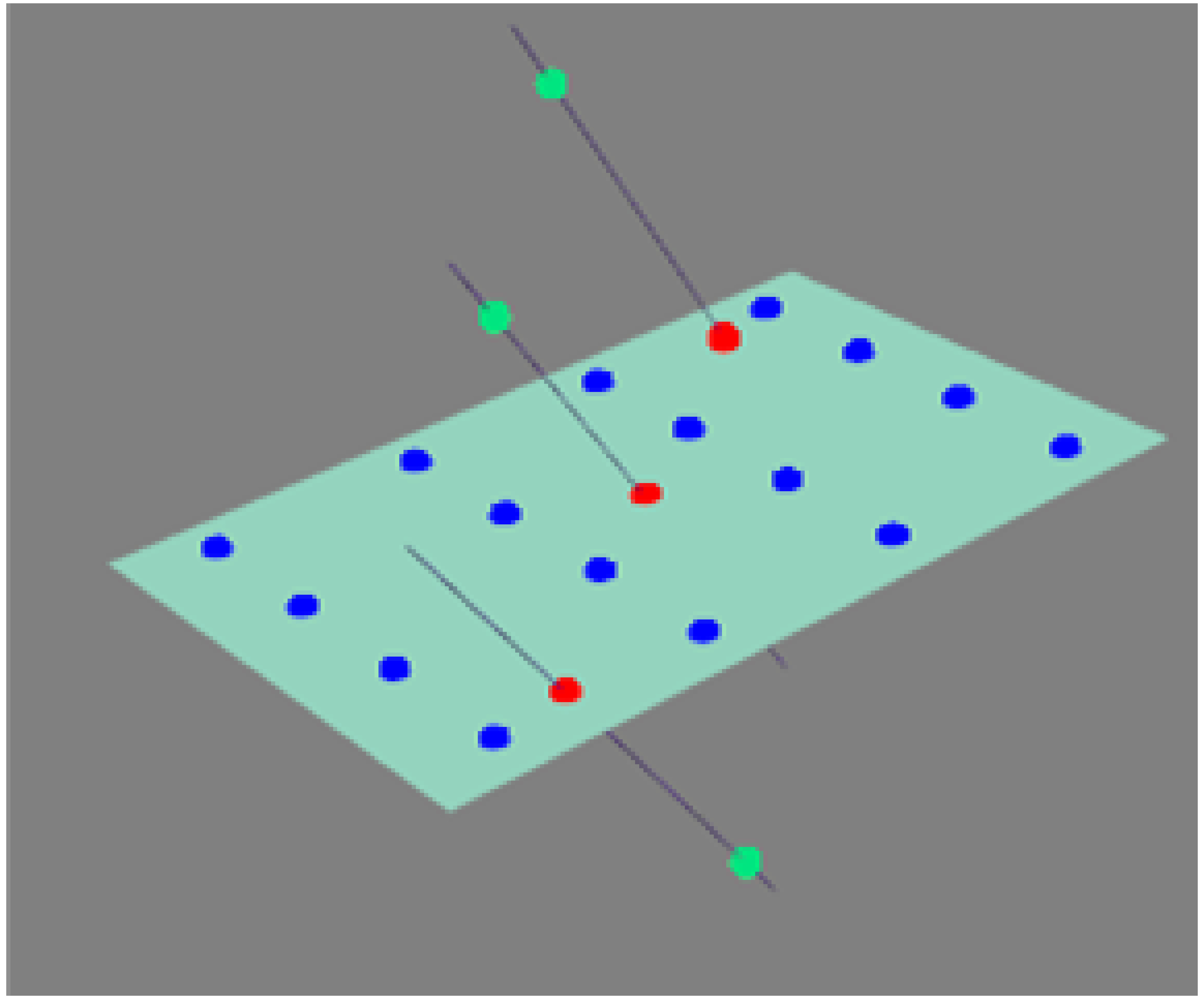
$$\boldsymbol{G} = \boldsymbol{G}(\theta^{el}, \varphi^{el}, \theta^{p}, \varphi^{p})$$

- ***S*** is vector of SECS amplitudes



SECS representation of E3D electric field

- The SECS system is set up at 300 km
- Map only E3D measurement points to 300 km
- Calculate the SECS geometry matrix
- Map the geometric matrix to E3D measurement points, which we call it ***G***



Fitting the SECS amplitudes to LoS data

The measured V_{los} by a receiver r is projection of $\boldsymbol{V} = \boldsymbol{V}_{\perp} + V_{\parallel}\,\hat{\boldsymbol{b}}$

$$\boldsymbol{V_{lr}} = \boldsymbol{K_r}\,\boldsymbol{V}_{\perp} + \boldsymbol{K_r}\,\hat{\boldsymbol{b}}\,V_{\parallel} + \boldsymbol{\varepsilon}_r,$$

$$\boldsymbol{V}_{\perp} = \frac{\boldsymbol{E}\times\boldsymbol{B}}{|\boldsymbol{B}|^2}$$ and $$\boldsymbol{E}.\boldsymbol{B} = \boldsymbol{0}$$

$$\boldsymbol{V_{lr}} = \boldsymbol{A}_{\perp r}\begin{bmatrix}\boldsymbol{E}_e\\ \boldsymbol{E}_n\end{bmatrix} + \boldsymbol{A}_{\parallel \boldsymbol{r}}\,\boldsymbol{V}_{\parallel} + \boldsymbol{\varepsilon}_r$$

$$\begin{bmatrix}\boldsymbol{E}_e\\ \boldsymbol{E}_n\end{bmatrix} = \boldsymbol{GS}$$

$$\boldsymbol{V_{lr}} = \boldsymbol{A_r}\begin{bmatrix}\boldsymbol{S}\\ \boldsymbol{V}_{\parallel}\end{bmatrix} + \boldsymbol{\varepsilon}_r$$

Fitting the SECS amplitudes to LoS data

$$\boldsymbol{V_{l1}} = \boldsymbol{A_1} \begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{V_{\parallel}} \end{bmatrix} + \boldsymbol{\varepsilon}_1$$

$$\boldsymbol{V_{l2}} = \boldsymbol{A}_2 \begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{V_{\parallel}} \end{bmatrix} + \boldsymbol{\varepsilon}_2$$

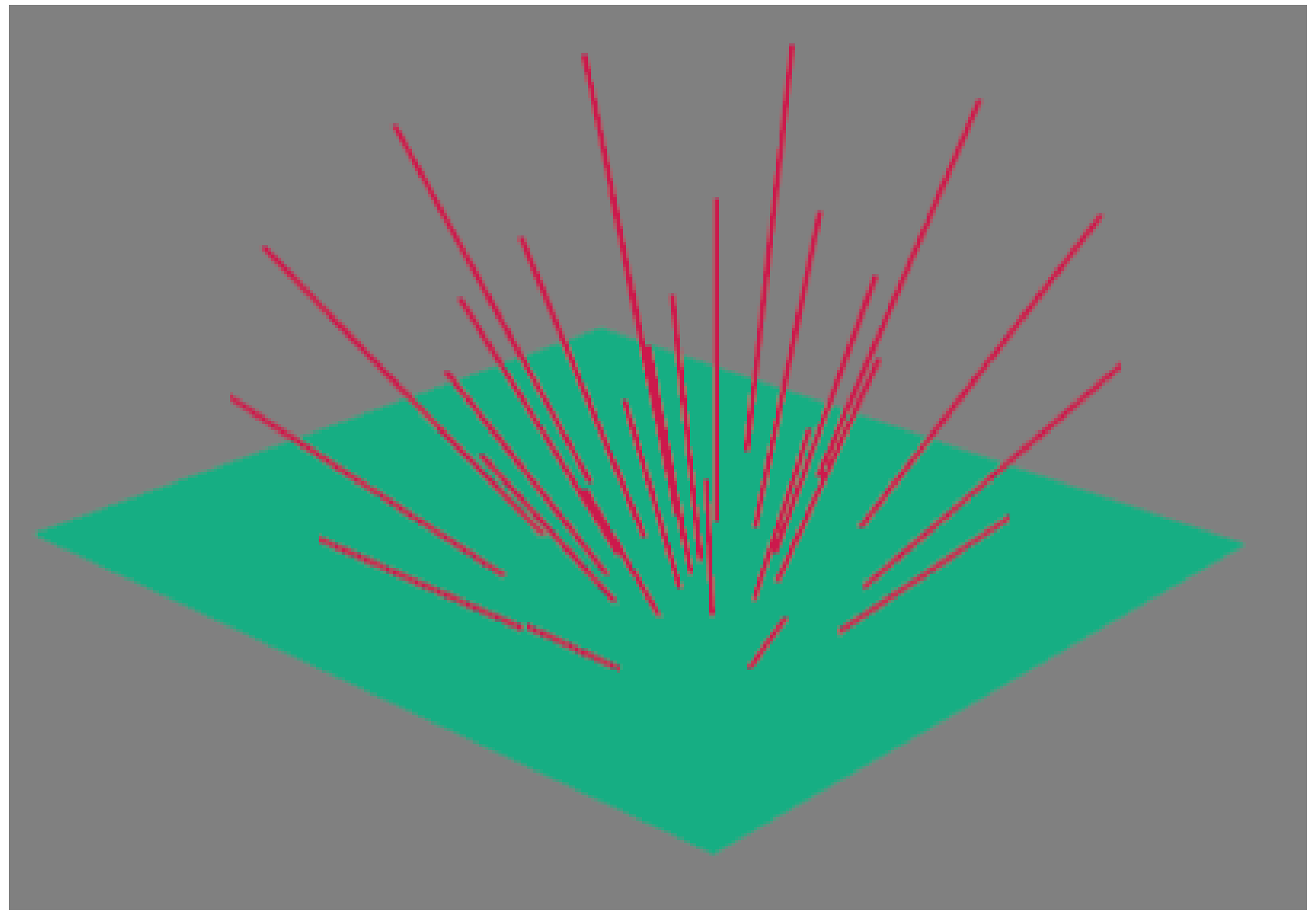
$$\boldsymbol{V_{l3}} = \boldsymbol{A_3} \begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{V_{\parallel}} \end{bmatrix} + +\boldsymbol{\varepsilon}_3$$

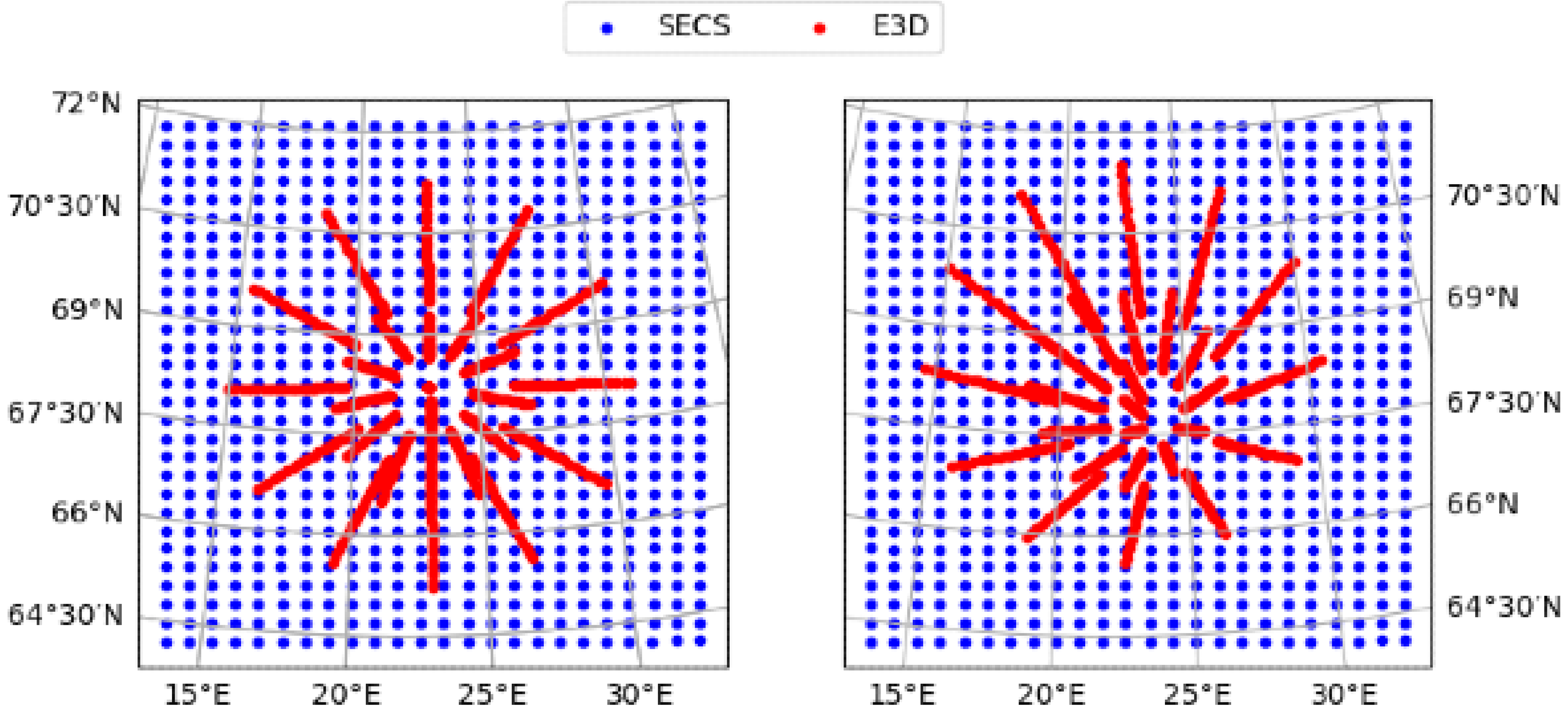
$$\boldsymbol{d} = \begin{bmatrix} \boldsymbol{V_{l1}} \\ \boldsymbol{V_{l2}} \\ \boldsymbol{V_{l3}} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{S} \\ \boldsymbol{V_{\parallel}} \end{bmatrix} + \boldsymbol{\varepsilon}$$

$$\begin{bmatrix} \tilde{\boldsymbol{S}} \\ \tilde{\boldsymbol{V}}_{\parallel} \end{bmatrix} = \left(\boldsymbol{A^T}\, \Sigma_{\boldsymbol{d}}{}^{-1} \boldsymbol{A} + \boldsymbol{\lambda^2 I}\right)^{-1} \boldsymbol{A^T}\, \Sigma_{\boldsymbol{d}}{}^{-1} d$$

$$\begin{bmatrix} \tilde{\boldsymbol{E}}_{\boldsymbol{e}} \\ \tilde{\boldsymbol{E}}_{\boldsymbol{n}} \end{bmatrix} = \boldsymbol{G}\tilde{\boldsymbol{S}}$$

SECS grid and E3D beam





One of the Cps suggested by Ogawa

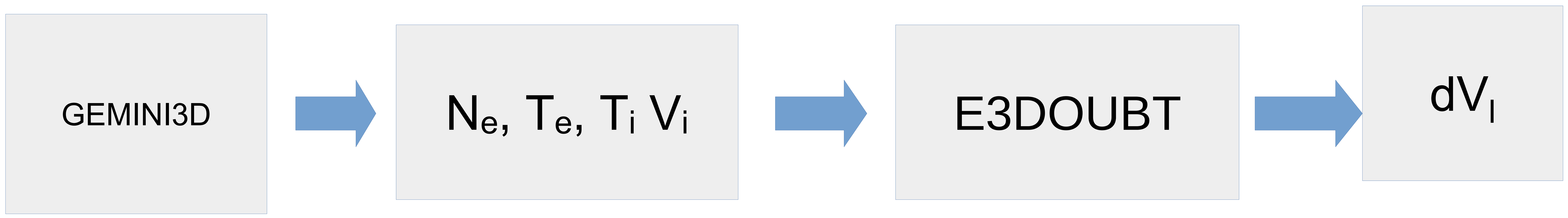
27 beams and 5 km pulse length

5 min total integration period,

~ 11 s: per beam integration period

GEMINI simulation and E3DOUBT error estimation

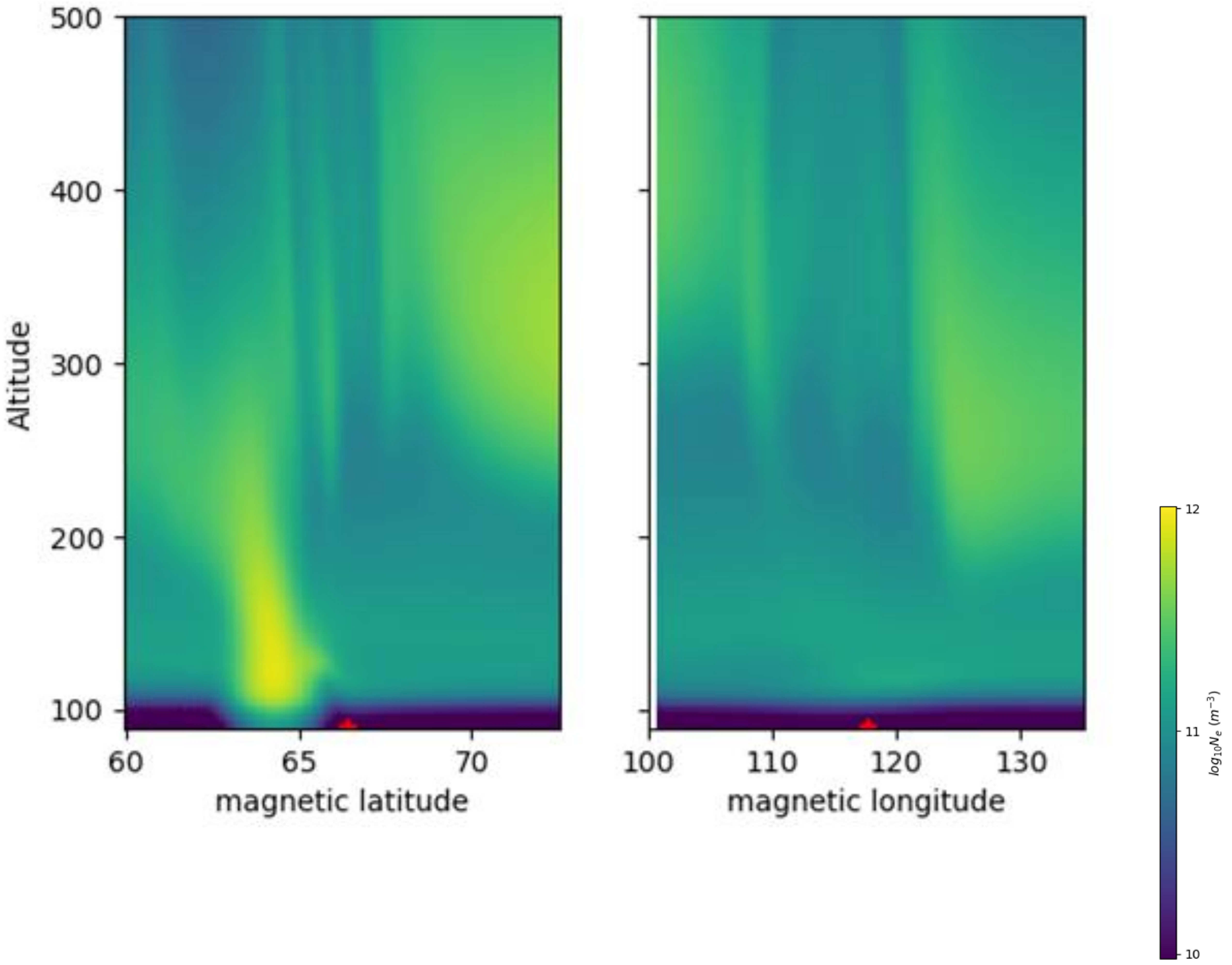
- We demonstrated our modeling method using simulated plasma parameters
- GEMINI3D takes electron precipitation energy flux and FAC as inputs and provides N_e, T_e, T_i and V_i
- The errors in the measured ion velocity are then estimated from E3DOUBT



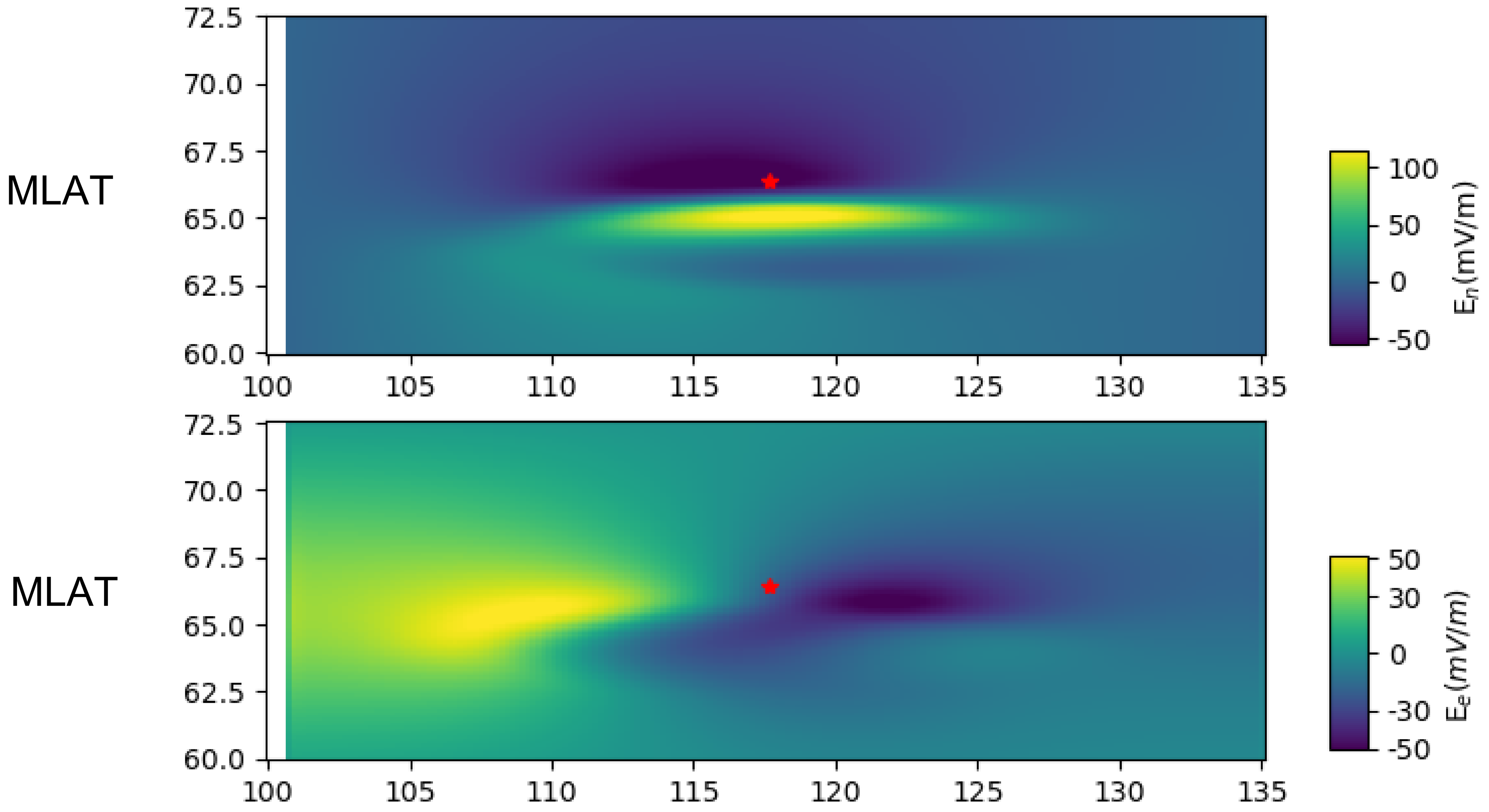
- Measured $\boldsymbol{V_l} = \boldsymbol{K}\,\boldsymbol{V}_i + \boldsymbol{e}$
- Where, $\boldsymbol{e} \sim N(\boldsymbol{0}, \boldsymbol{dV_l^2})$, and $\boldsymbol{K}$ is the projection matrix to the radar line of sights

GEMINI simulation data

Electron density



Electric field at 300 km

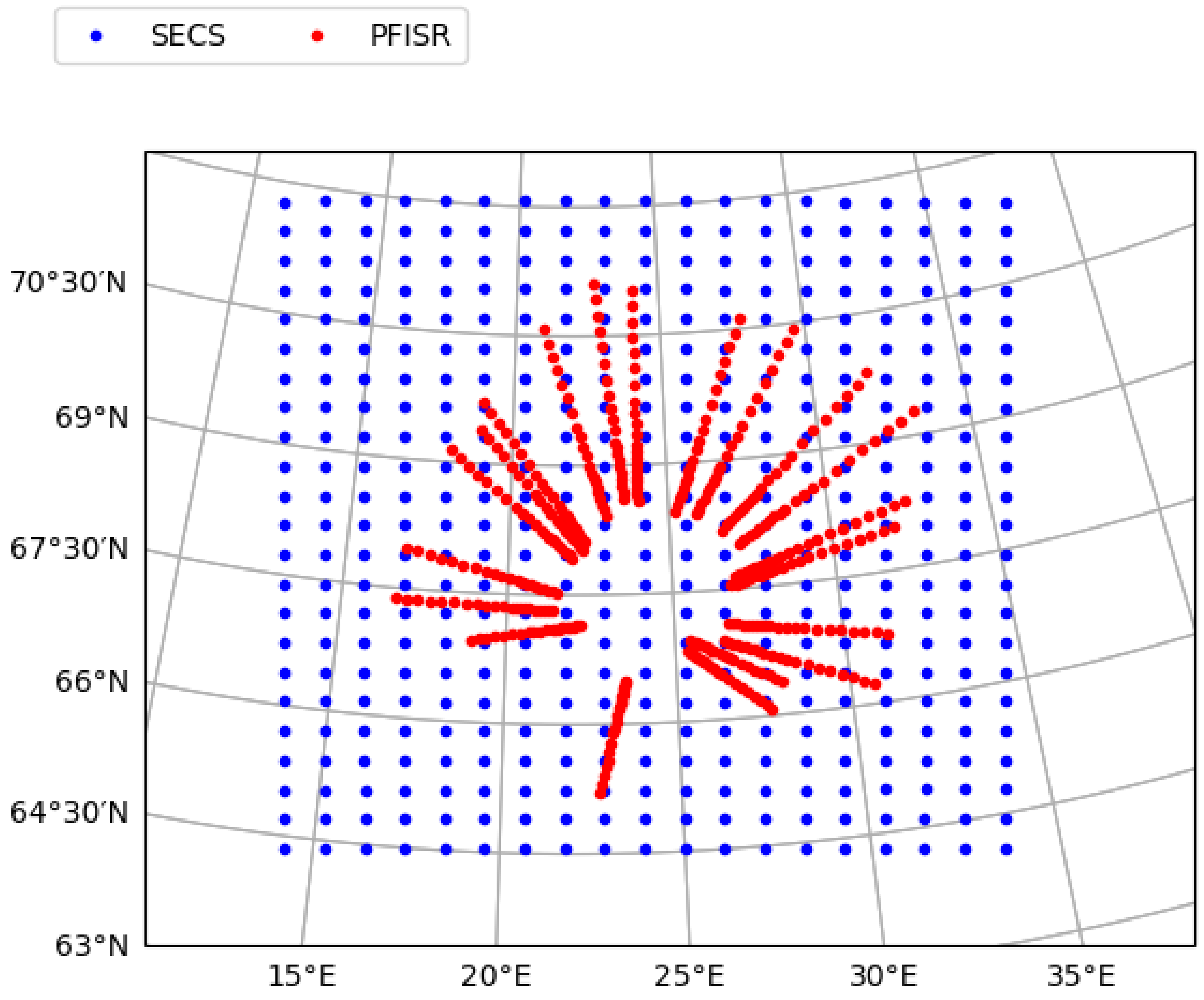


SECS analysis of tristatic E3D electric field: 5 min integration period

SECS analysis of tristatic E3D electric field: 1 min integration period

Monostatic fit with PFISR beam

- Low elevation (<70) beams are used
- high elevation beam $\boldsymbol{V_l}$ data dominated by noise.
- Number of beams: 21
- Pulse length: 5 km
- Total Integration period: 5 min
- Integration period per beam: ~ 7 s

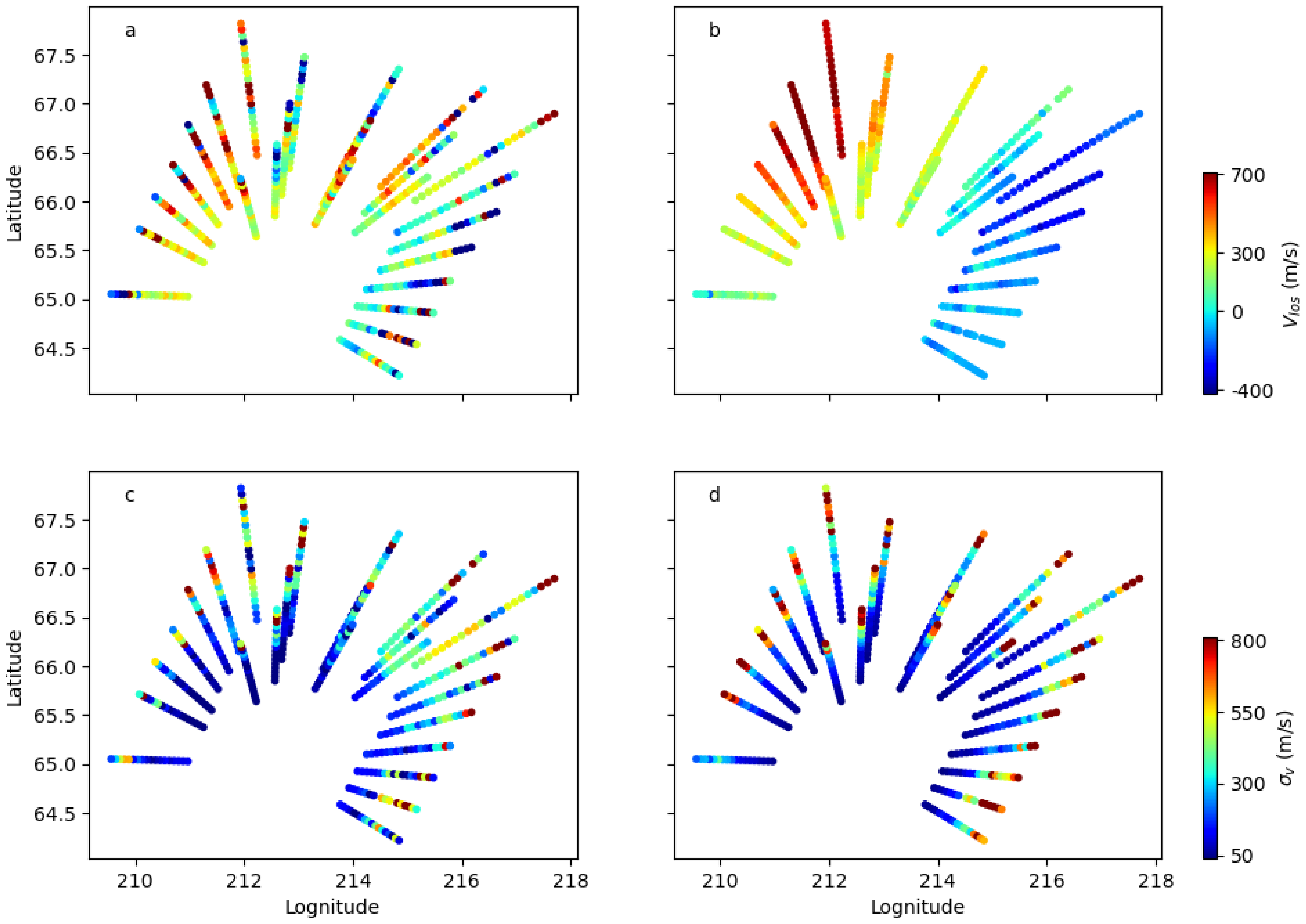


Monostatic LoS fit with PFISR beam

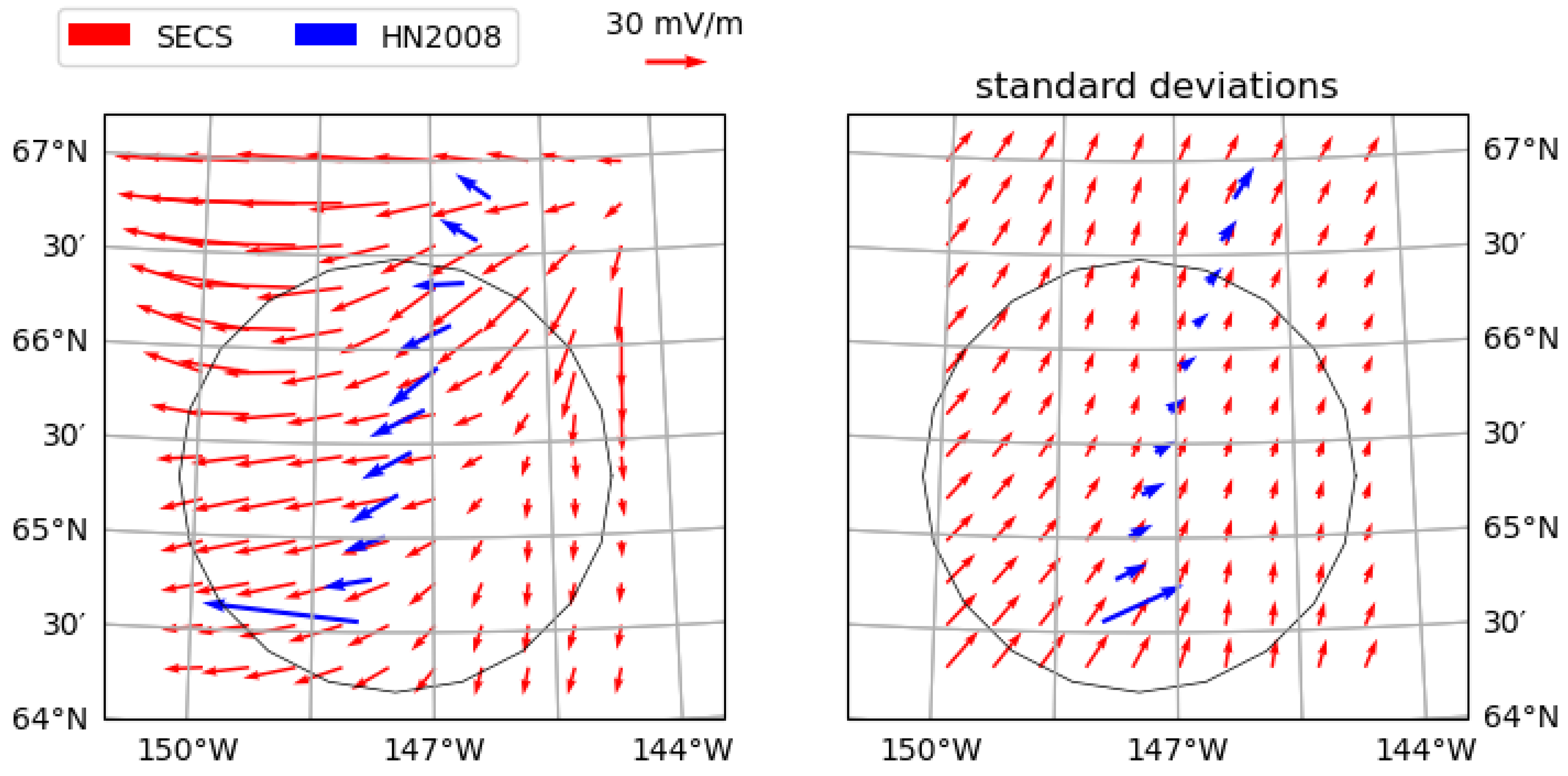
$$V_{l1} = A_1 \begin{bmatrix} S \\ V_{\parallel} \end{bmatrix} + \varepsilon_1$$

Monostatic electric field fit

Monostatic LoS fit with real PFISR data



Monostatic Electric field fit with real PFISR data



Summary

- We developed a method to fit regional model of electric field to E3D data
- We tested the method using synthetic data based on GEMINI3D simulation and E3DOUBT realistic error estimation
- Our method captures all essential features of the true background electric field when applied to the tristatic E3D configuration.
- The monostatic version of the model is applied to real PFISR data, and the model gives reasonable 2D variation of the electric field.