Complex system approach to geospace and climate studies

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Outline of a talk

- Importance of complex system approach
- Phase space reconstruction
- Recurrence plot analysis
- Test for determinism
- Examples

Complex approach

- Motivation: Most ”real-life” systems are too complicated to be described directly by fundamental laws

- Useful in systems with many degrees of freedom, in the absence of thermodynamic equilibrium (open,dissipative systems), in non-stationary, nonlinear systems

- Morphology is more important than microscopic structure (universality)

- Order emerges spontaneously (self-organisation)

Examples:
Chaos, Phase transitions (first and second order), self-organized criticality

Phase space

- All possible states are represented (each possible state of the system corresponding to one unique point in the phase space). For mechanical systems, the phase space usually consists of all possible values of position and momentum variables.

- Questions:

- *How trajectory evolves in the phase space: do trajectories recur (come back) to "same part" of the phase space?*
- *Does the recurrence have a period (periodic system)?*
- *Do trajectories diverge exponentially (chaotic system) or with a power law (stochastic system)?*
- *Are trajectories parallel in the same box of the phase space (deterministic system)?*

Phase space reconstruction

- Consider a time series **s(t)** whose length is **N**
- Embedding dimension is **m** and **τ** is a time delay
- Time delay embedding for t=1, 2,..., N-(**m**-1)**τ** :

$$\vec{X}_t = (s_t, s_{t+\tau}, s_{t+2\tau}, \ldots, s_{t+(m-1)\tau})$$

m-dimensional vector
for time series **s**

τ is the first zero in the autocorrelation function

- For example, for **τ**=2 and **m**=3:

$$\vec{X}(t_1) = \left(s(t_1), s(t_3), s(t_5)\right)$$

$$\vec{X}(t_2) = \left(s(t_2), s(t_4), s(t_6)\right)$$

$$\vec{X}(t_3) = \left(s(t_3), s(t_5), s(t_7)\right)$$

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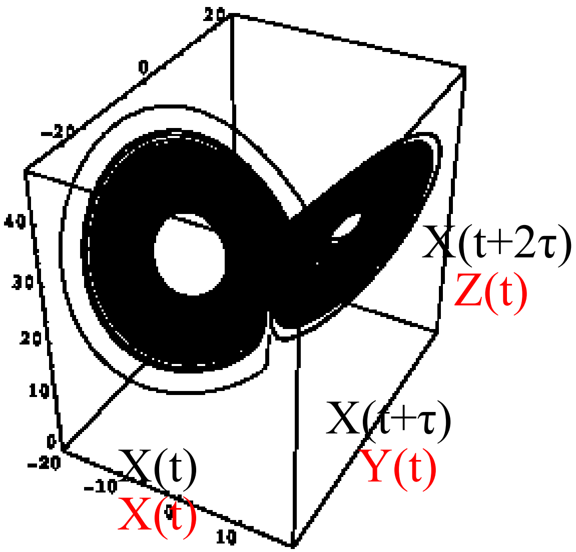
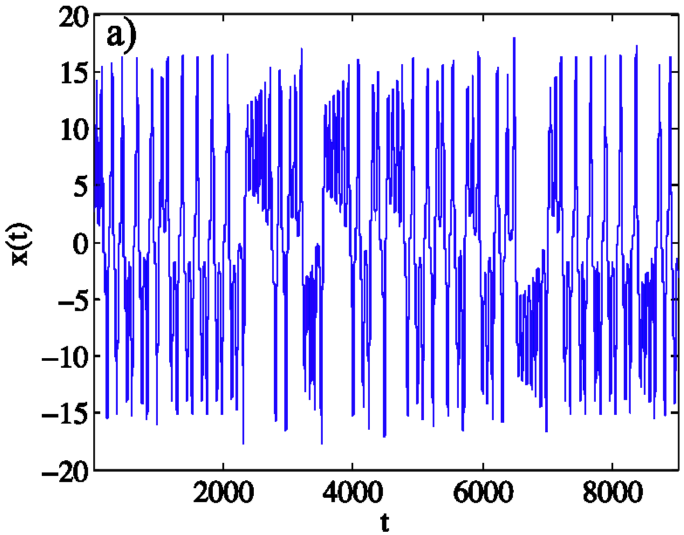
- For the Lorenz system:

$$\{x(t), y(t), z(t)\} \Leftrightarrow \{x(t), x(t+\tau), x(t+2\tau)\}$$

(example of Lorenz attractor)





- Model of thermal convection in the atmosphere

$$\frac{dx}{dt} = a(y - x)$$

$$\frac{dy}{dt} = -xz + cx - y$$

$$\frac{dz}{dt} = xy - bz$$

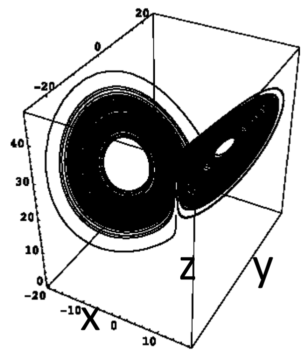
- Fixed points:

$$x^* = y^* = z^* = 0$$

$$x^* = y^* = \pm\sqrt{b(c-1)}, z^* = c - 1$$

a=10, b=8/3, and c=28

- Recurrence plots (RPs) visually represent recurrences of the trajectories in the phase space. Suppose we have a trajectory $\{\vec{x}_i\}_{i=1}^{N}$ of a system in the phase space.



$$\frac{dx}{dt} = \sigma(y - x)$$

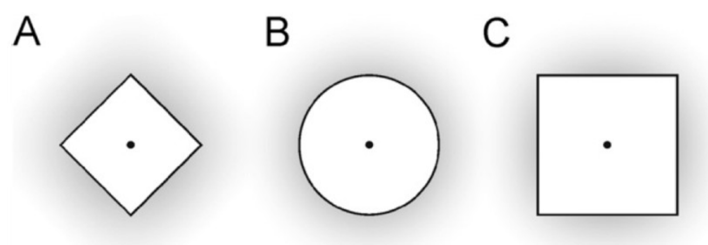
$$\frac{dy}{dt} = -xz + rx - y$$

$$\frac{dz}{dt} = xy - bz$$

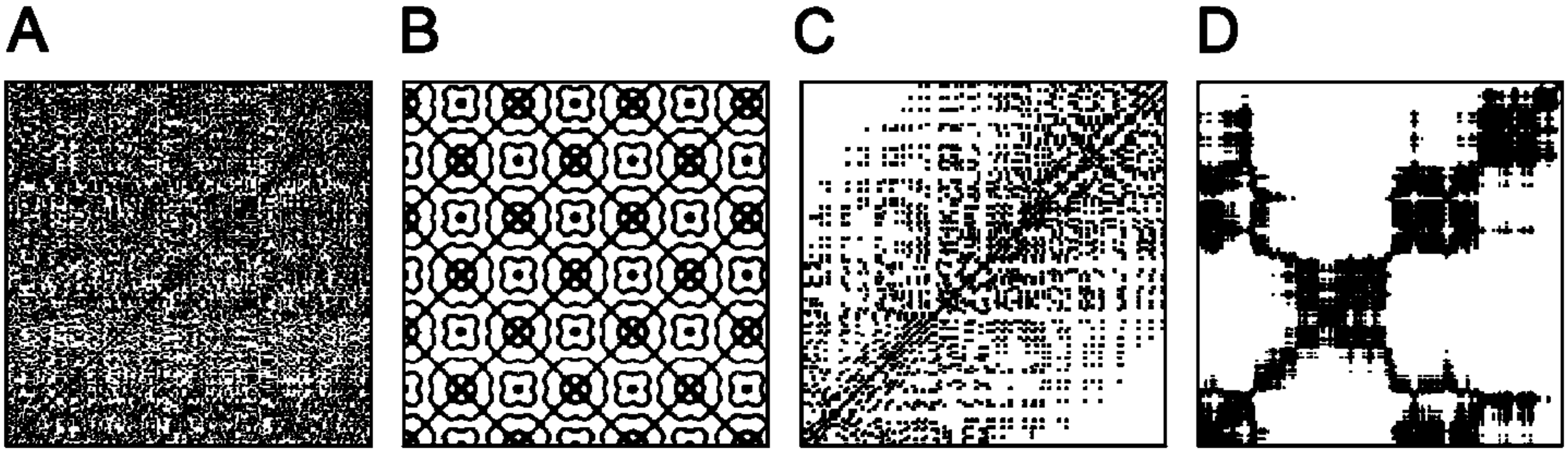
RP matrix *R* consists of zeros and ones; when trajectories recur:

$$\left|\vec{x}_i - \vec{x}_j\right| < \varepsilon \quad , \; R_{i,j}(\varepsilon) = 1 \;.$$

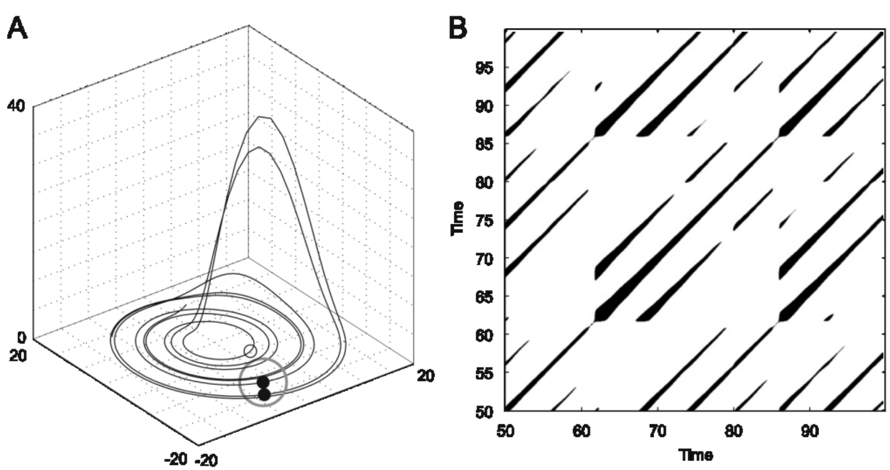
Here, ε is a threshold defined to obtain a fine structure of the RP.



Neighborhood around a point (A): L_1 norm, (B): L_2 norm, (C): L_∞ -norm



A-uniformly distributed white noise
B-superposition of harmonic oscillators
C-logistic map corrupted with linearly increasing term
D-Brownian motion

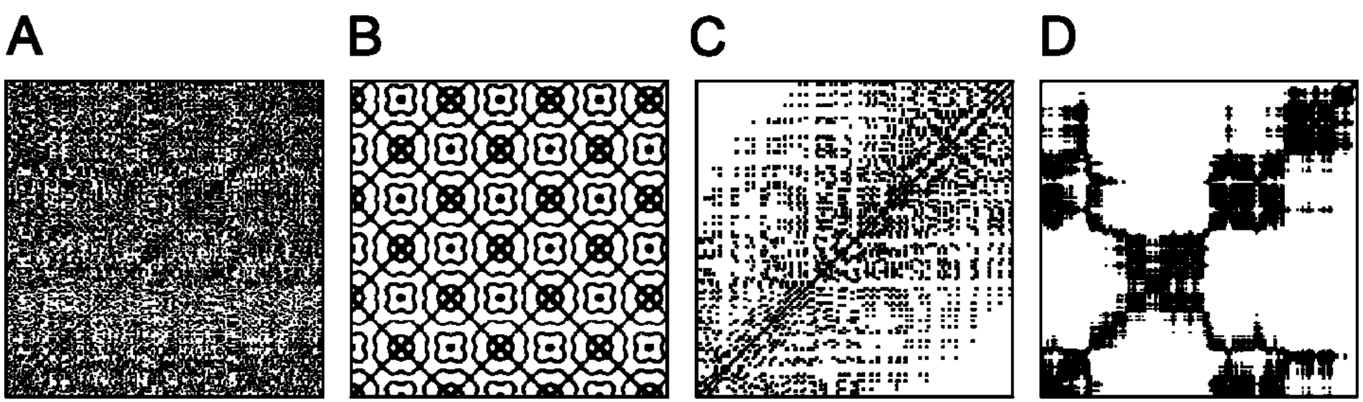


(A)-segment of the phase space trajectory of the RÖssler attractor

(B)-corresponding recurrence plot

RP texture…

- **Homogenous RPs**: stationary systems, where relaxation times are short in comparison with the time spanned by the RP
- **Periodic and quasiperiodic** systems have RPs with diagonal lines
- **A drift** can be seen in non-stationary systems, where RP pales away from the min diagonal
- **Abrupt changes** in the dynamics as well as extreme events cause white areas or bands in the RP



Recurrence quantification analysis (RQA)

1. Measures based on vertical lines and recurrent point density

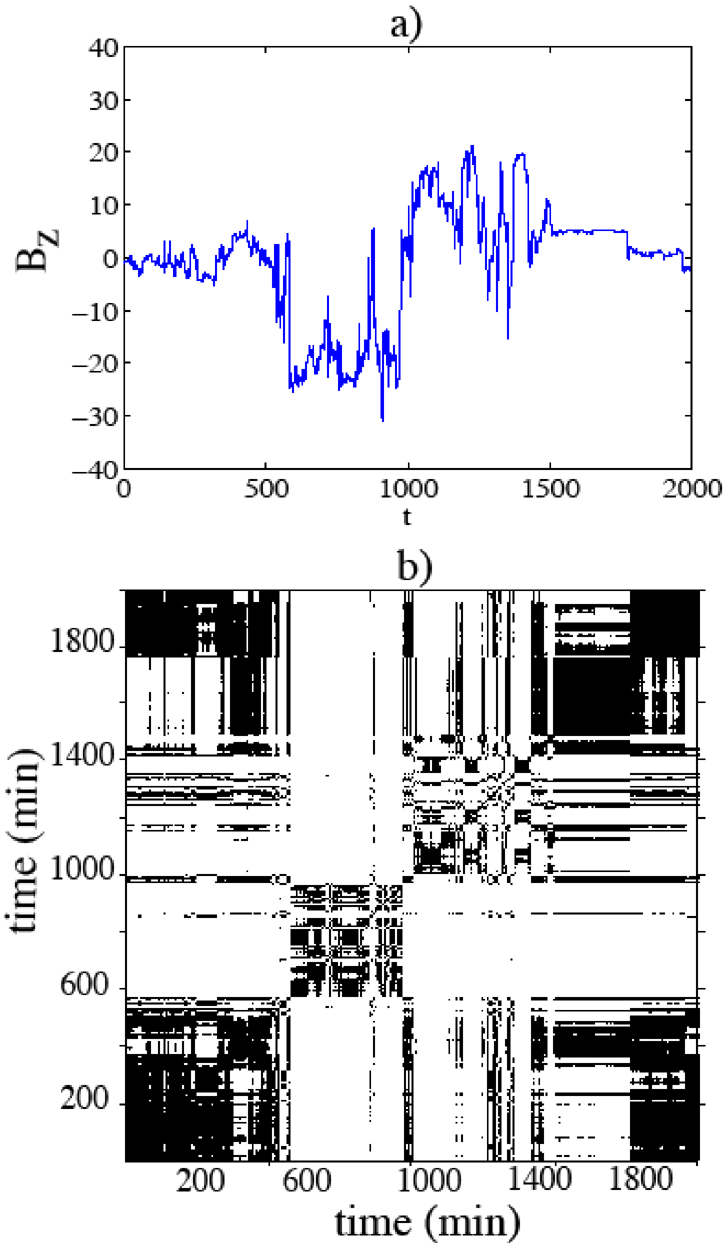
2. <u>Measures based on diagonal lines :</u>

Histogram P(ε,l) is defined:

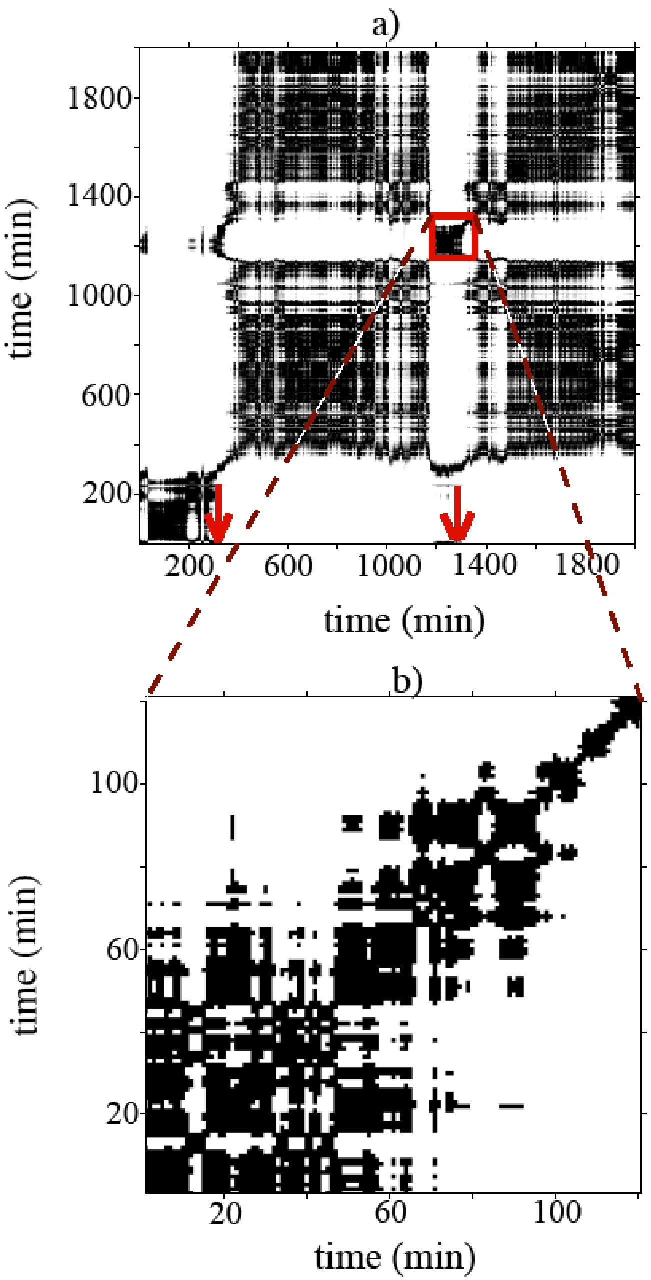
$$P(\varepsilon, l) = \sum_{i,j=1}^{N} \left(1 - R_{i-1,j-1}(\varepsilon)\right)\left(1 - R_{i+1,j+1}(\varepsilon)\right) \prod_{k=0}^{l-1} R_{i+k,j+k}(\varepsilon).$$

$$l_{max} = \max\left(\{l_i\}_{i=1}^{N_l}\right) \quad , \quad N_l = \sum_{l \geq l_{min}} P(l) \quad .$$

$$\langle l^- \rangle = \frac{\sum l^{-1} P(l)}{\sum P(l)}$$



RP for IMF Bz
Storm on 6th of April, 2000

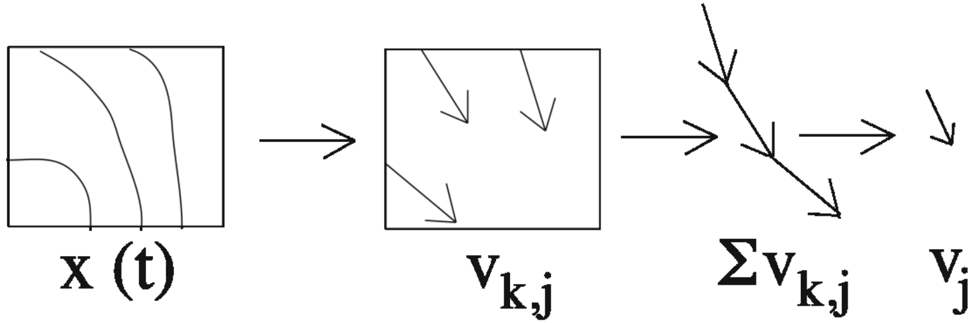


RP for AE index

(two substorm onsets at
5:01 and 21:26)
a)whole day

a)second substorm

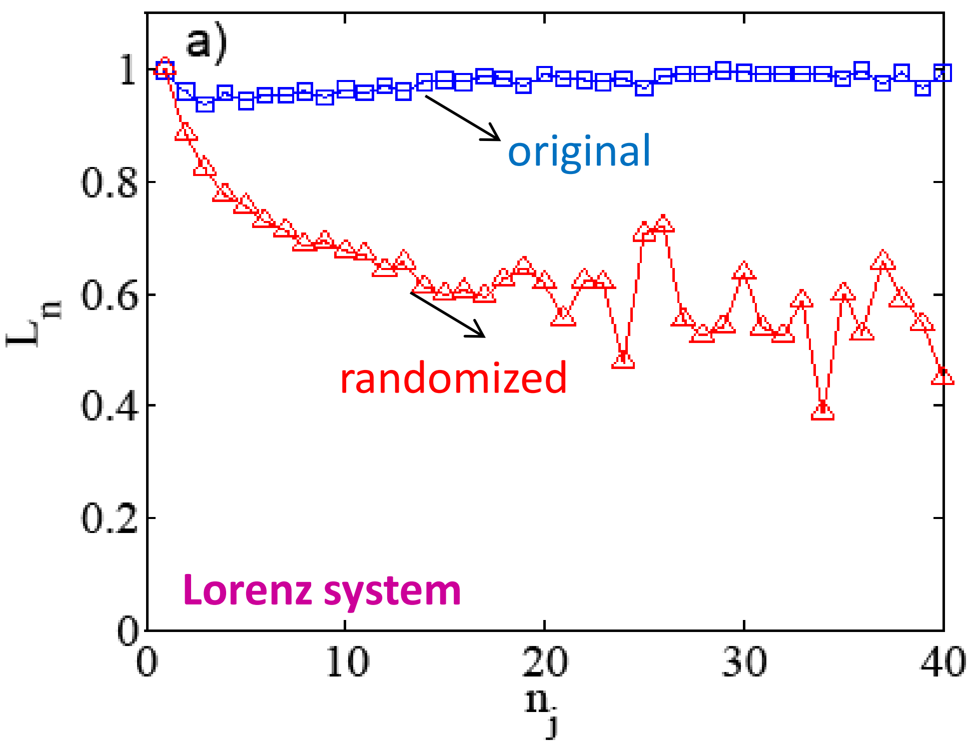
Test for determinism



$$\frac{dx}{dt} = f(x)$$

$$\Delta\vec{x}(t) = \vec{x}(t+b) - \vec{x}(t)$$

b=time spent in the box

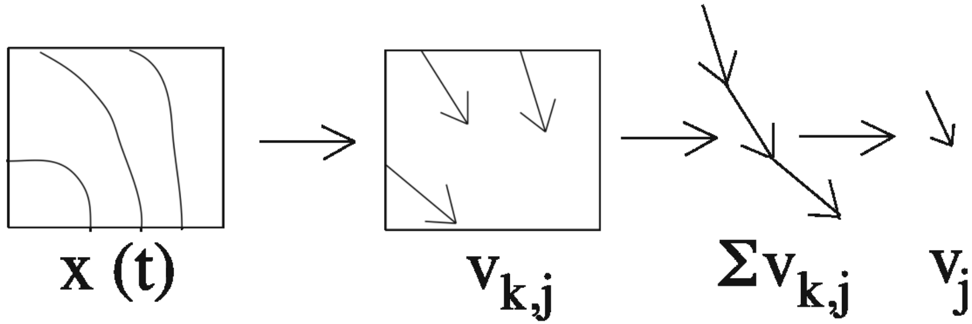


$$V_j = \frac{1}{n_j}\sum_{k=1}^{n_j}\frac{\Delta x_{k,j}}{\left|\Delta x_{k,j}\right|}$$

$$L_n \equiv \left\langle V_j \right\rangle_{n_j = n}$$

n_j =number of passes

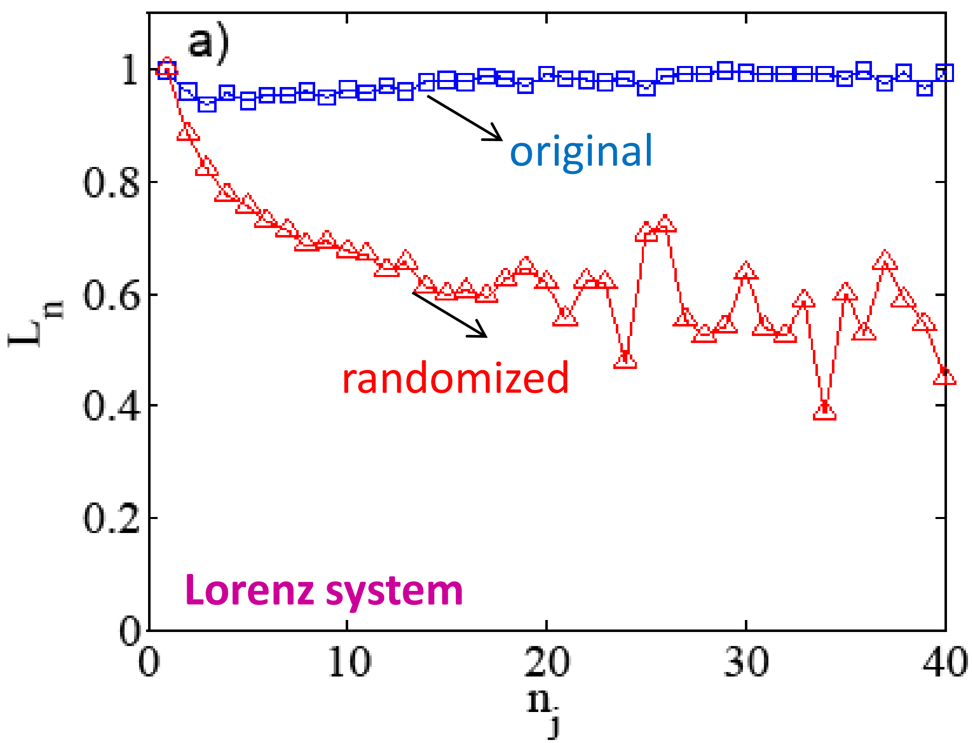
Test for determinism (Kaplan and Glass, 1992)



$$\frac{dx}{dt} = f(x) + w$$

$$\Delta\vec{x}(t) = \vec{x}(t+b) - \vec{x}(t)$$

b=time spent in the box



$$V_j = \frac{1}{n_j}\sum_{k=1}^{n_j}\frac{\Delta x_{k,j}}{|\Delta x_{k,j}|}$$

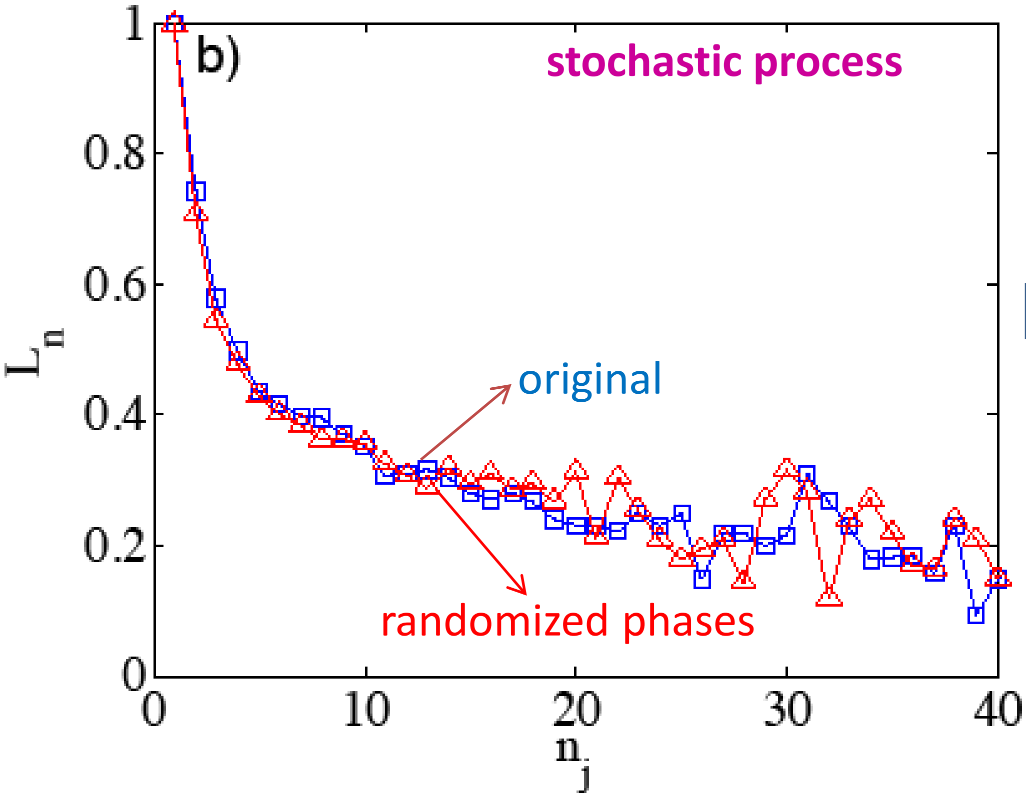
$$L_n \equiv \langle V_j \rangle_{n_j = n}$$

n_j =number of passes

(Phases are randomized in Fourier space)

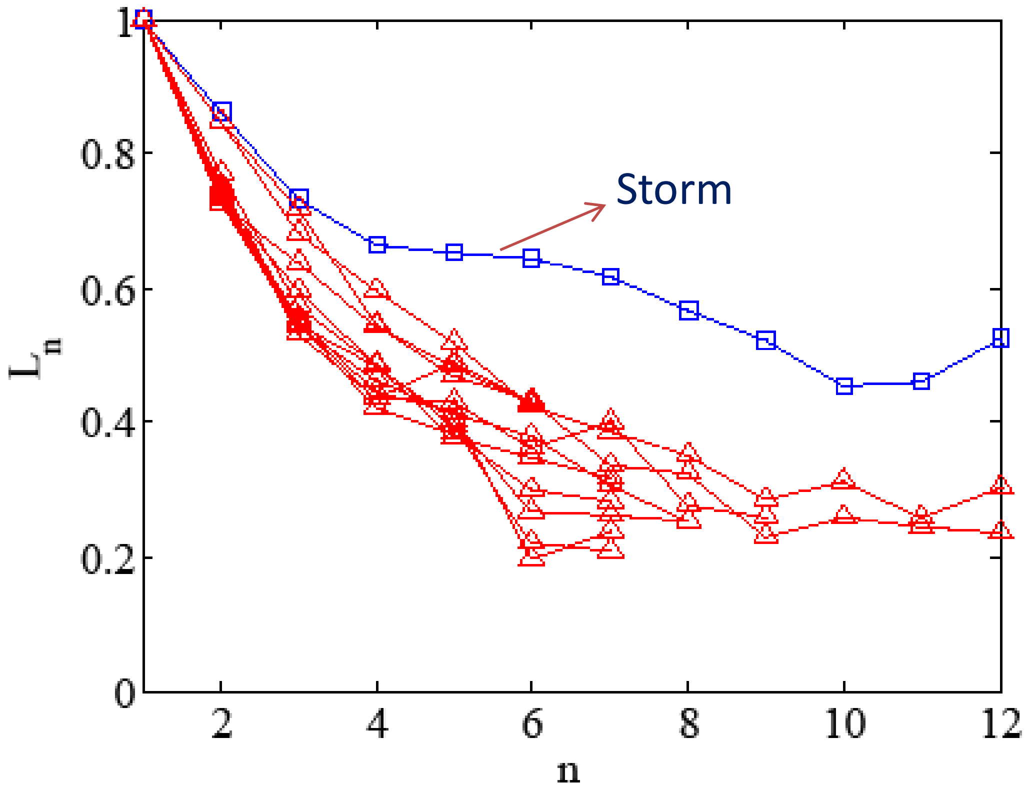
When randomized L_n vs. N is under the one for the original signal

Process has low-dimensional and nonlinear component



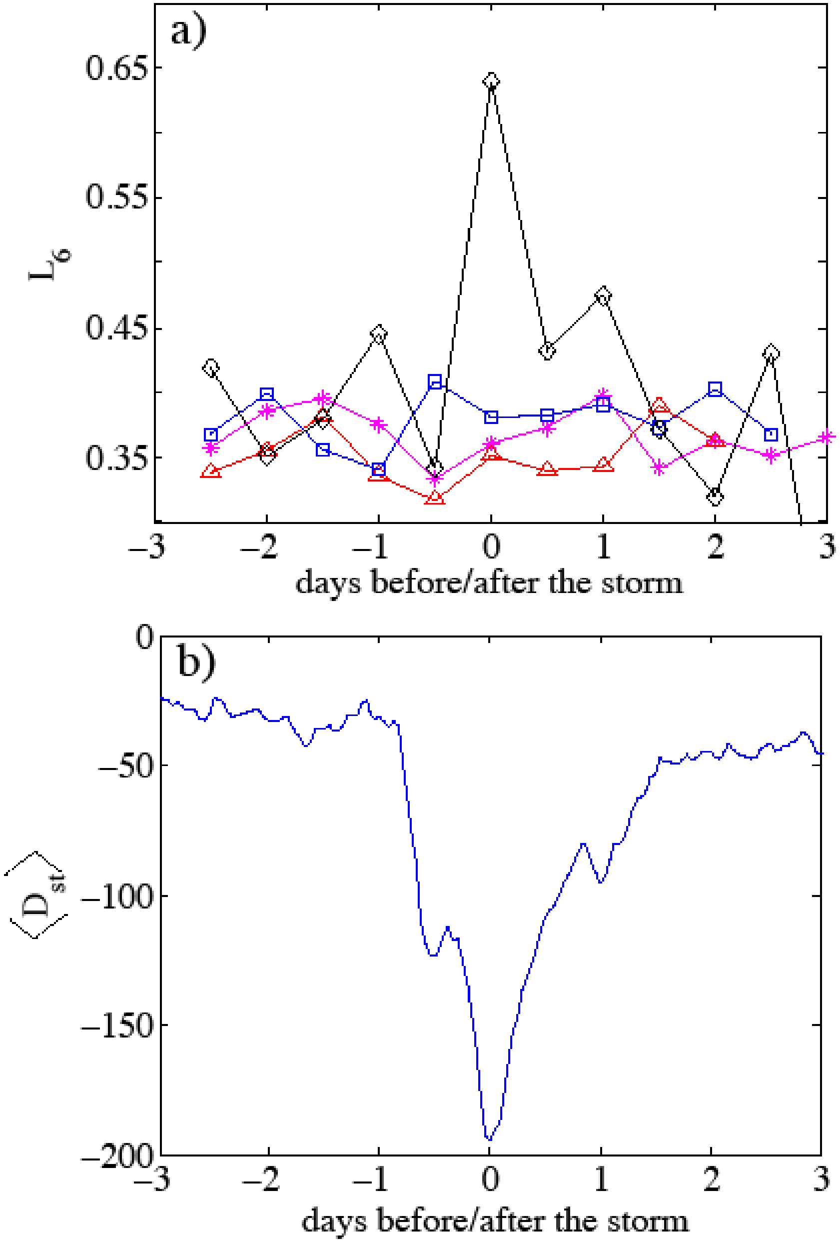
Neither
low-dimensional
nor
nonlinear
component

Examples…..



Increased determinism during the storm

L_n vs. N for
SYM-H data averaged over 10 storms,
for the period 3 days before/ after the storm's main phase with a resolution of 12 hours



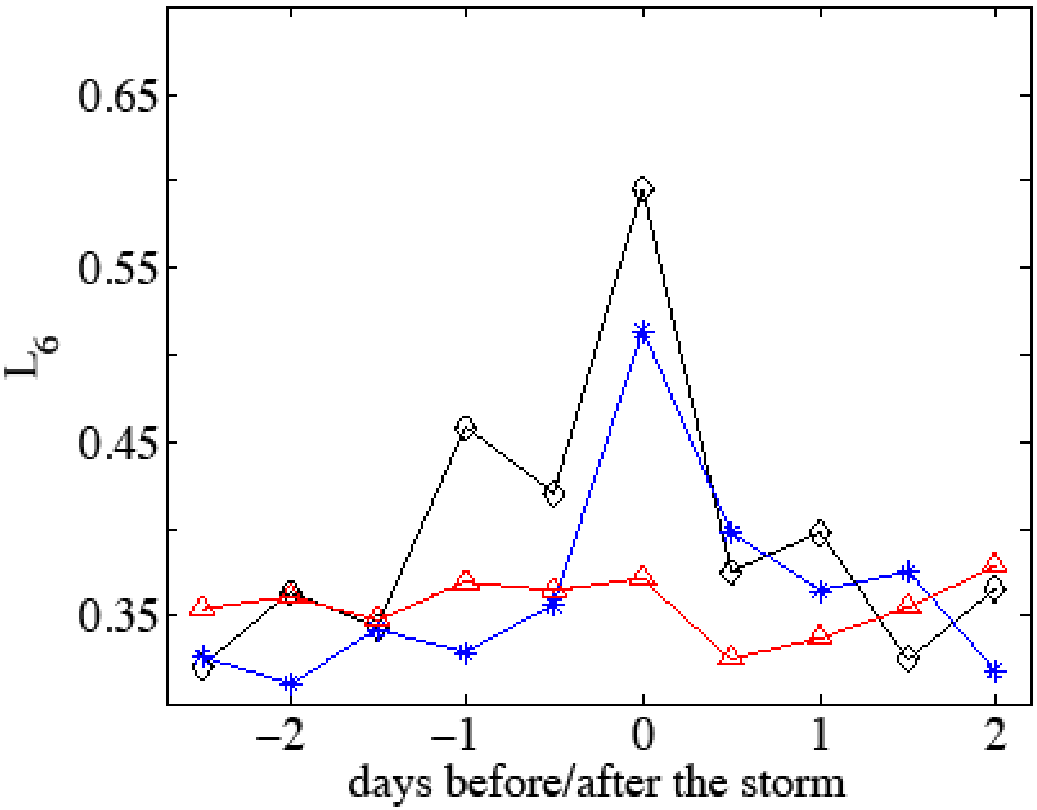
◇ SYM-H

□ IMF B_z

△ Flow speed V

* Linear stochastic process (fitted from SYM-H through linear square regression)

L_6 averaged over 10 storms, for period 3 days before/ after the storm's main phase, with a resolution of 12 hours



SYM-H

SYM-H*

sqrt (P_{dyn})

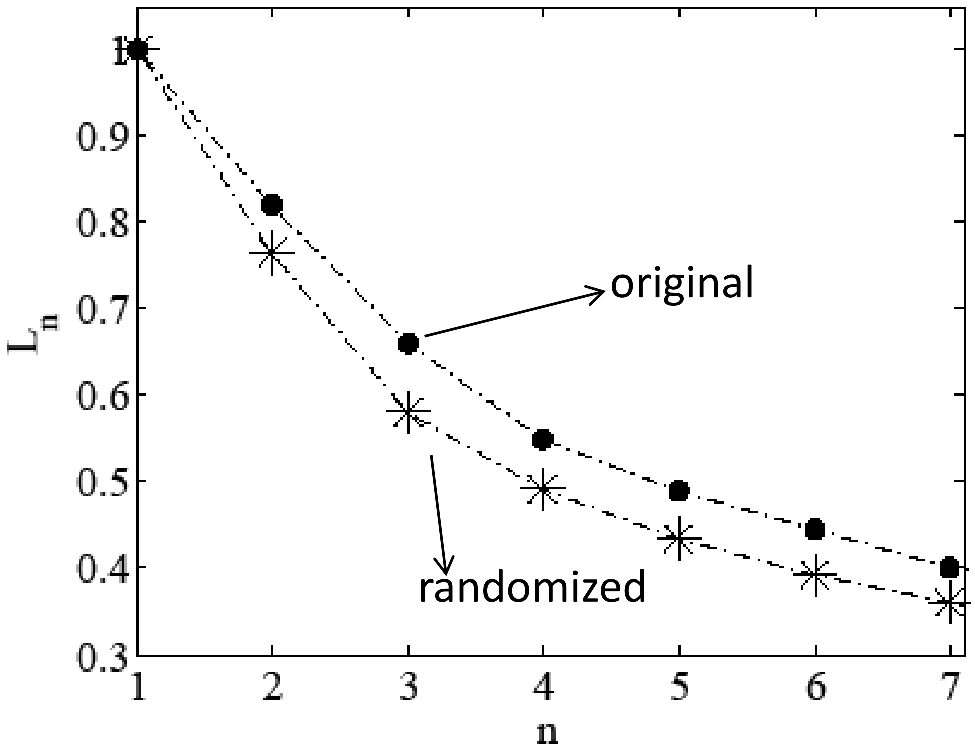
SYM-H* has also increased determinism during storm

Exclude contribution from magnetopause current:

$$\mathrm{SYM-H}^{*} = 0.77(\mathrm{SYM-H}) - 11.9\sqrt{P_{dyn}}$$

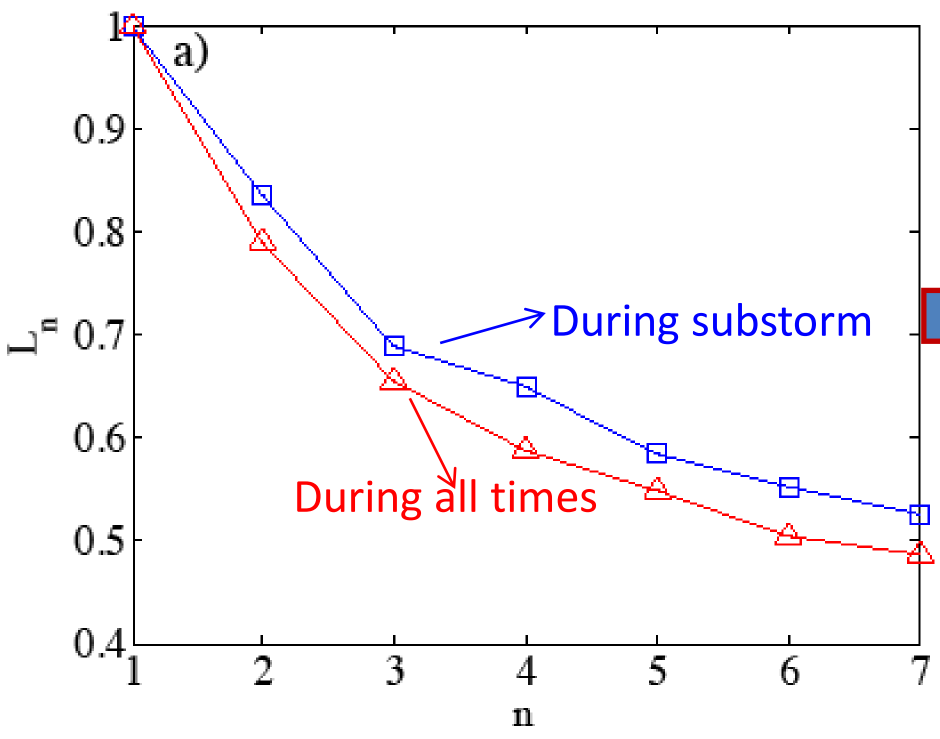
P_{dyn} is Solar wind (dynamical) pressure

Determinism in AE index



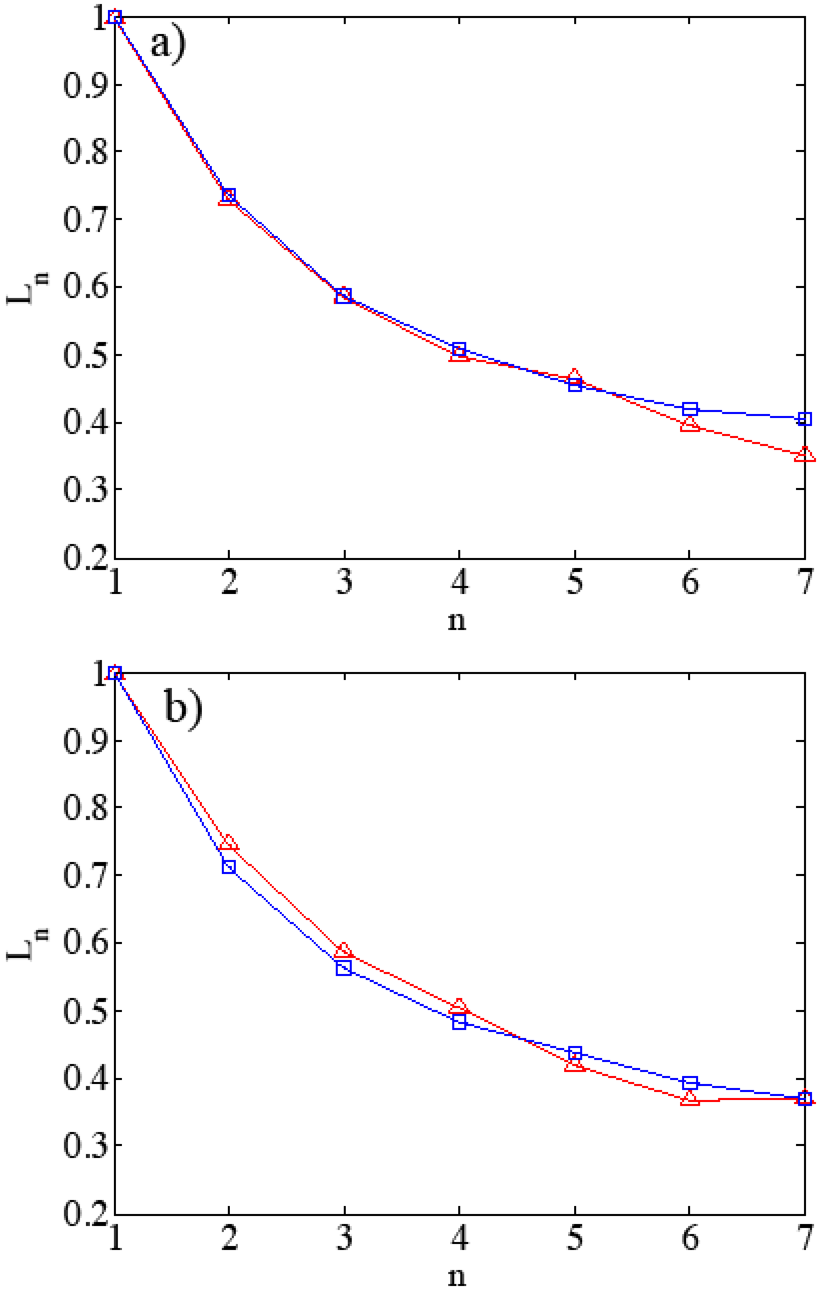
AE index has low-dimensional, nonlinear component (the same result is obtained for AL, AU and PC index)

Mean L_n over substorms
(database from Frey & Mende, 2002)



AE index
is more deterministic
during substorms

The same is shown for **AU**
and **PC** (polar cup) index



a) B_z

b) V

□ during substorms

△ during all times