Tentamen för kursen Rymdfysik (1FA255)
2018-10-18

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.

Time: 08:00 - 13:00

Allowed tools: Beta Mathematics Handbook, Nordling & Österman Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.

The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. This part is only graded by pass/fail. You do not need to solve problem number N if you passed the corresponding examlet number N this year.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so. Note: The text in italics is true.
   i. Communication satellites are usually put in a polar geostationary orbit.
   ii. To increase the apoapsis of a satellite, one should use the onboard thrusters at periapsis.
   iii. The mass of a rocket (including fuel) scales logarithmically with the velocity increase it provides.
   iv. *As you are writing this exam, final preparations are going on for the launch of BepiColombo at 01:45 UT this Saturday from Kourou in French Guyana. The final destination is Mercury.* As it is to go inward in the solar system, the spacecraft will leave the Earth in the direction opposite Earth's motion around the Sun.
   v. The Ariane V launcher used for BepiColombo is a multistage rocket. The main advantage with this is that with several stages, the exhaust gas velocity increases.

   (b) If a cylindrical spacecraft with its height equal to its radius has a certain temperature at Earth orbit (1 AU), what ratio of height to radius would be needed to attain the same temperature at Mercury orbit (0.3 AU)? All other properties of the spacecraft are assumed equal, and in both cases one of the circular ends points to the Sun.

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Plasmas are expected to be electrically neutral on length scales greater than the Debye length.

ii. Plasma recombination is typically fast, except in the presence of a dense background of neutral gas, where it is much slower.
iii. In ideal MHD, no electrical currents can flow through the plasma.
iv. In an inertial frame, the solar wind plasma is seen to flow along the Parker spiral.
v. Electrical currents flow at the magnetopause.

(b) A static flow solution for the solar wind velocity profile $v(r)$ can be obtained by solving

$$(v^2 - v_{\mathrm{T}}^2)\frac{1}{v}\frac{dv}{dr} = 2\frac{v_{\mathrm{T}}^2}{r} - \frac{GM}{r^2}.$$

Draw a curve qualitatively showing the solution $v(r)$ appropriate to the solar wind, marking the point where the RHS$= 0$. What physical quantity is represented by v_T?

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. Drift motions (e.g., gradient-B, ExB, ...) would not be observed in a plasma in which consisted purely of electrons and positrons (opposite charges, same masses)
ii. Only a force with a component parallel to the magnetic field will induce a drift motion in a plasma.
iii. The $\mathbf{E} \times \mathbf{B}$ drift does not cause a current to flow
iv. The magnetic moment of a gyrating particle is an adiabatic invariant, and is conserved if the field changes slowly relative to the gyro motion period
v. A particle trapped in the dipolar magnetic field of the Earth, with constant kinetic energy, cannot cross the equator during its motion.
vi. The gyroradius of a particle must be smaller than the Debye length

(b) Describe, using a diagram, the motion of positively charged ions and negatively charged electrons in a uniform, steady background magnetic field aligned with the Z direction, assuming that the both species have a pitch angle $\alpha = 45°$? If the magnetic field strength is slowly increased by a factor of two, what happens to the motion of the particles?

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. The aurora is mainly caused by solar wind ions hitting the ionosphere.
ii. Magnetospheric substorms are associated with reconection in the photosphere.
iii. An ionosphere will form around any planet with an atmosphere orbiting a star emitting ionizing radiation.
iv. The main reason for ionization in the Earth's ionosphere is the high temperarure of the atmosphere at high altitude.
v. The Pedersen and Hall conductivities each have a maximum value at some altitude, while the parallel conductivity increases with altitude.

(b) Figure 1 shows some space weather data for the first three months of this year. Except the bottom plot, all are from a spacecraft at the Earth-Sun L1 Lagrange point.

i. There seems to be some periodic structures, most clearly seen in the solar wind speed. What phenomenon on the Sun may give rise to these?
ii. Why is the same periodicity seen also in the Kp index (which quantifies geomagnetic activity)? What do you think may happen to the magnetosphere during the periods of high solar wind speed?
iii. The highest Kp index is found a few days before the end of the plot interval. Still, neither the solar wind density nor velocity seem to be higher at that time than in some of the other peaks. Can you suggest some other mechanism or feature explaining the Kp maximum on this day?

Your complete answer to (b) should probably be between ten lines and one page in length, plus any figures you may wish to draw.

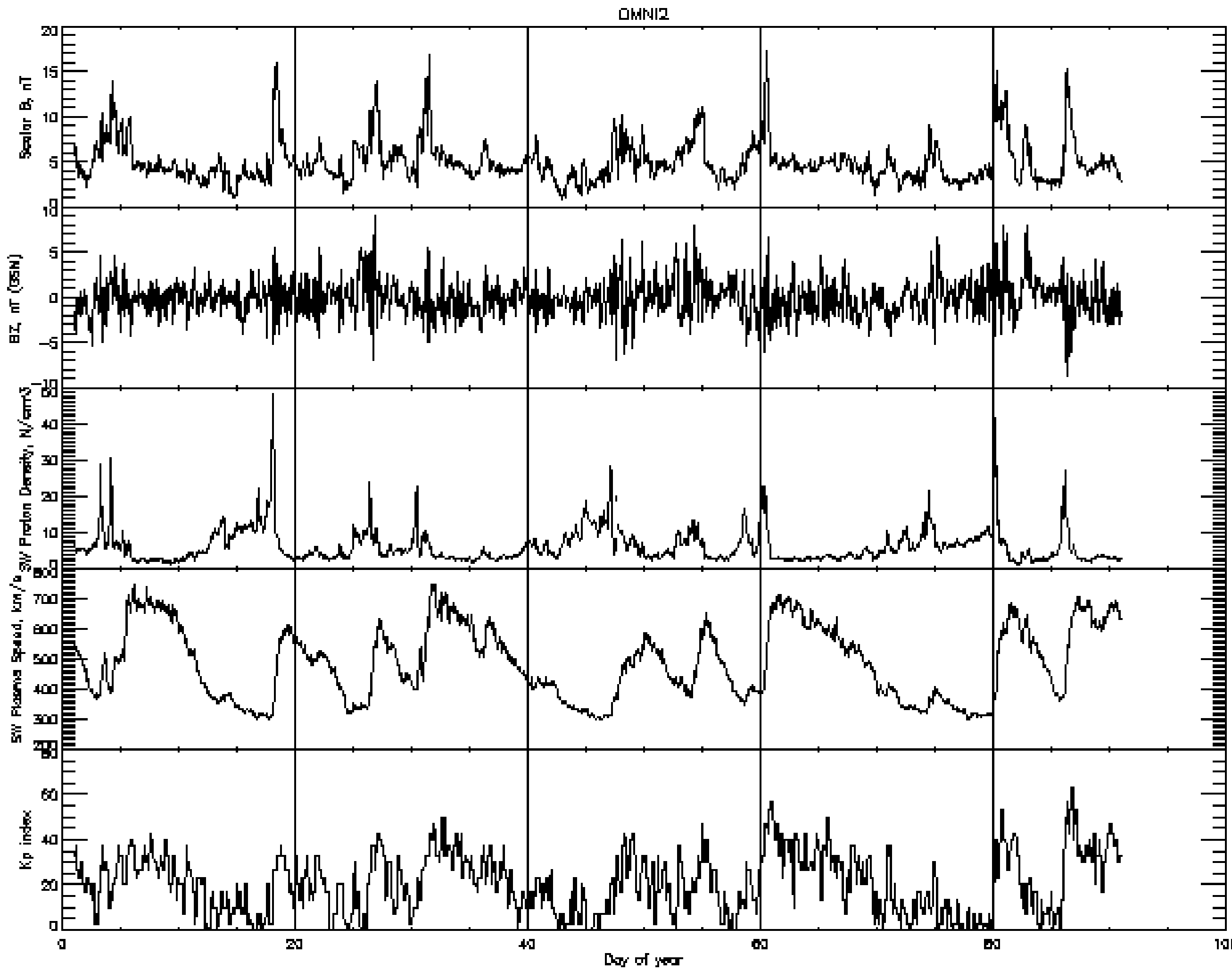


Figure 1: Space weather data for January-March 2018. Plots from top to bottom: (i) strength and (ii) z component of the interplanetary magnetic field [nT], (iii) solar wind proton number density [cm^{-3}], (iii) solar wind flow speed [km/s], and Kp index (multiplied by 10).

Part B

5. The Parker Solar Probe was launched a couple of months ago for the closest inspection of the Sun and the corona ever undertaken. Figure 2 shows the route of the spacecraft.

   (a) At Earth orbit, typical values for the solar wind speed, plasma number density, and magnetic field strength can be taken to be 400 km/s, 5 cm^{-3} and 5 nT, respectively. What typical values of the same parameters would you expect Parker Solar Probe will see when closest to the Sun (0.04 AU)? (2 p)

   (b) Does the total energy of the trajectory increase or decrease at the first Venus flyby? (1 p)

   (c) Given your answer to (b), what do you think the spacecraft trajectory looks like in the frame of reference of Venus? Draw a zoomed-in figure of the neighbourhood of Venus, perhaps 10 or 20 Venus radii on each side, with the planet at the centre and the sun far away to the right. Sketch the spacecraft trajectory as it flies by the planet, with arrows indicating the direction of motion. Do not worry about getting distances and angles numerically correct: it is the general nature of the trajectory which is of interest, particularly the incoming and outgoing directions. Motivate your sketch in words. (2 p)

6. (a) Draw a sketch of the Earth's magnetosphere, viewed from the dusk side (so that the Sun is to the left of the page). Label the following regions, boundaries and features: Earth's magnetic field, magnetopause, bow shock, magnetosheath, magnetosphere, and stream lines of the solar wind. Show the direction of the major electric currents that flow *through* the page, and indicate the locations where magnetic reconnection can be important. (3 p)

   (b) By assuming a balance between the magnetic pressure of the Earth's magnetic field, and the dynamic (ram) pressure of the solar wind, calculate an expression for the distance to the magnetopause at the sub-solar point. During intervals of extreme space weather, the solar wind velocity may increase by a factor of 10 - what effect does this have on the size of the magnetosphere? (2 p)

   (c) In a plane perpendicular to that used above, sketch the current system that supports the magnetic field configuration in the magnetotail. (1 p)

7. Consider an idealised description of the magnetic field in the Earth's current sheet in which $\mathbf{B} = [B_x(z), 0, 0]$, where

$$B_x(z) = \begin{cases} B_0 & z \geq a \\ B_0 \left(\frac{z}{a}\right)^3 & -a < z < a \\ -B_0 & z \leq -a \end{cases}$$

   with $a = 3000$ km and $B_0 = 10$ nT.

   (a) Evaluate the current density and the magnetic force density in each part of the region defined. (2 p)

   (b) The rate of electric energy released per unit time and volume is $\mathbf{j} \cdot \mathbf{E}$. Using the equations of ideal MHD, $\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$ and $\rho \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B}$, show that all the energy released from the field is converted to kinetic energy of the plasma. *Hint:* $\mathbf{v} \cdot \frac{d\mathbf{v}}{dt} = \frac{d}{dt}(v^2)$. (1 p)

8. (a) Approximating the Earth's magnetic field with a dipole,

$$\mathbf{B}(r,\theta) = -B_E \left(\frac{R_E}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

show that the equation of a terrestrial magnetic field line is given by $r = r_0 \sin^2\theta$, and derive an expression for the magnetic field magnitude $|B|$ along the field line as a function of altitude $h = r - R_E$. (2 p)

(b) Consider the 'bounce' motion of a charged particle along a dipolar field line: state which two quantities may be conserved for charged particles moving within such a field, defining all relevant terms, and show therefore why

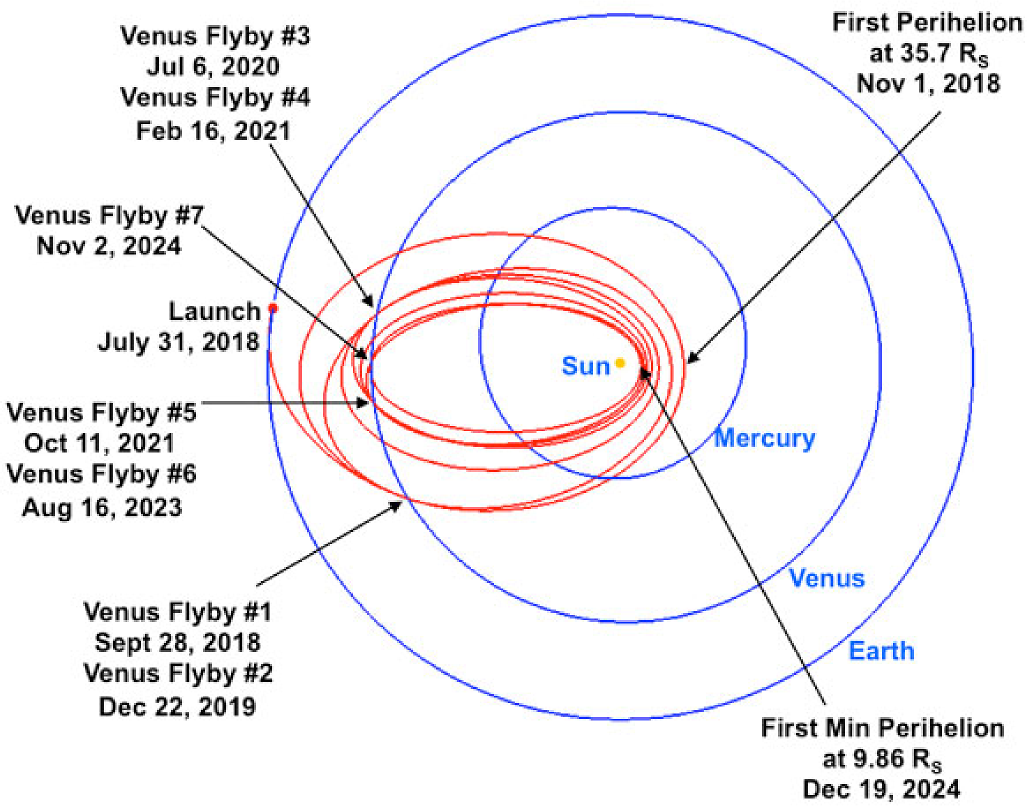
$$\frac{\sin^2\alpha}{B} = \text{constant},$$

where $\alpha = \arctan(v_\perp / v_\parallel)$. (1 p)

(c) Plasma with a Maxwellian (isotropic) velocity distribution is present in the equatorial magnetosphere. What fraction of this overall distribution is able to reach down to altitudes of 300 km above Uppsala, latitude 60°? (3 p)

(Hint: consider the solid angle of the loss-cone for this field line.)

Lycka till!



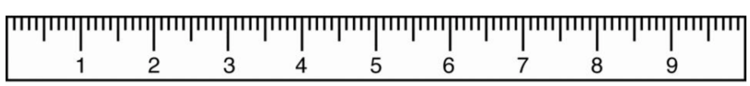


Figure 2: Path of the Parker Solar Probe through space. Image source: NASA.

Space Physics Formulas:
Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \mathrm{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s (\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\,\mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_{\mathrm{dyn}} = \frac{1}{2} n m v^2$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_{\mathrm{P}} & -\sigma_{\mathrm{H}} & 0 \\ \sigma_{\mathrm{H}} & \sigma_{\mathrm{P}} & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_{\parallel} \end{pmatrix} = \sigma_{\mathrm{P}} \mathbf{E}_{\perp} + \sigma_{\mathrm{H}} \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B} + \sigma_{\parallel} \mathbf{E}_{\parallel}$$

Conductivities:

$$\begin{aligned} \sigma_{\mathrm{P}} &= \frac{ne}{B}\left(\frac{\omega_{\mathrm{ci}}\nu_{\mathrm{i}}}{\omega_{\mathrm{ci}}^2 + \nu_{\mathrm{i}}^2} + \frac{\omega_{\mathrm{ce}}\nu_{\mathrm{e}}}{\omega_{\mathrm{ce}}^2 + \nu_{\mathrm{e}}^2}\right) \\ \sigma_{\mathrm{H}} &= \frac{ne}{B}\left(\frac{\omega_{\mathrm{ci}}^2}{\omega_{\mathrm{ci}}^2 + \nu_{\mathrm{i}}^2} - \frac{\omega_{\mathrm{ce}}^2}{\omega_{\mathrm{ce}}^2 + \nu_{\mathrm{e}}^2}\right) \\ \sigma_{\parallel} &= ne^2\left(\frac{1}{m_{\mathrm{i}}\nu_{\mathrm{i}}} + \frac{1}{m_{\mathrm{e}}\nu_{\mathrm{e}}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Kepler's third law:

$$T^2 \propto a^3$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$

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