Exam in Antenna Theory

Time: 18 March 2010, at 8.00–13.00.
Location: Polacksbacken, Skrivsal

You may bring: Laboratory reports, pocket calculator, English dictionary, Råde-Westergren: "Beta", Nordling-Österman: "Physics Handbook", or comparable handbooks.

Six problems, maximum five points each, for a total maximum of 30 points.

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1. a) If the complex electric field is denoted $\mathcal{E}(\mathbf{r})$, find the corresponding instantaneous (time-dependent) electric field $\mathcal{E}(\mathbf{r},t)$. (1p)

   b) The array factor of a N-element uniform array can be written

   $$AF = \frac{\sin\left(\frac{N}{2}\psi\right)}{\sin\left(\frac{1}{2}\psi\right)},$$

   where $\psi = kd\cos\theta + \beta$ is the progressive (total) phase shift. Specify the condition for β for a

   i) broadside array; ii) end-fire-array; iii) phased (or scanning) array. (2p)

   c) A half-wavelength dipole has the input impedance $(73 + j42.5)\ \Omega$. What is the input impedance of a quarter-wavelength monopole placed directly above an infinite perfect electric conductor? (1p)

   d) A *folded* half-wavelength dipole has an input resistance of approximately

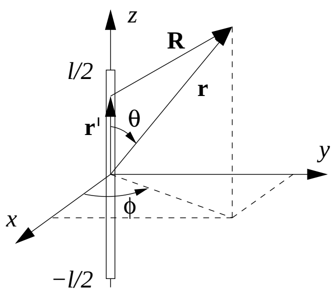
   i) 50 Ω; ii) 75 Ω; iii) 150 Ω; iv) 300 Ω; v) 600 Ω (1p)

2. Consider a very thin finite length dipole of length l which is symmetrically positioned about the origin with its length directed along the z axis according to the figure. In the far-field region the condition that the maximum phase error should be less than $\pi/8$ defines the inner boundary of that region to be $r = 2l^2/\lambda$. For $r \le 2l^2/\lambda$, we are in the radiating near-field region and the far-field approximation is not valid. By allowing a maximum phase error of less than $\pi/8$, show that the inner boundary of this region is at $r = 0.62\sqrt{l^3/\lambda}$.

   Hint: The vector potential is given by

   $$\mathbf{A} = \frac{\mu}{4\pi}\int \mathbf{I}_e(x',y',z')\frac{e^{-jkR}}{R}\mathrm{d}l' \, .$$

   Expand R, where the higher order terms become more important as the distance to the antenna decreases. Note that $\mathbf{r}' = z'\hat{z}$.



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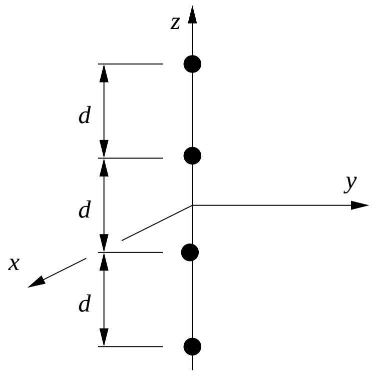
3. An infinitesimal horizontal electric dipole of length l and constant electric current I_0 is placed parallel to the y axis a height $h = \lambda/2$ above an infinite electric ground plane.

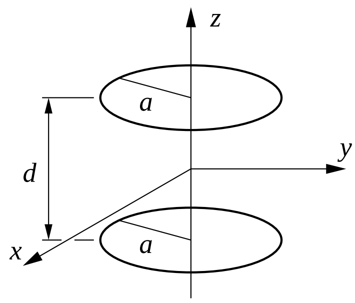
   a) Find the spherical **E-** and **H**-field components radiated by the dipole in the far-zone.
   b) Find the angles of all the nulls of the total field.

4. A four-element uniform array has its elements placed along the z axis with distance $d = \lambda/2$ between them according to the figure below.

   a) Derive the array factor and show that it can be written as $\dfrac{\sin(2\psi)}{\sin(\psi/2)}$, where ψ is the progressive phase shift between the elements.
   b) In order to obtain maximum radiation along the direction $\theta = 0^\circ$, where θ is measured from the positive z axis, determine the progressive phase shift ψ.
   c) Find all the nulls of the array factor.



5. Two identical constant current loops with radius a are placed a distance d apart according to the figure below. Determine the smallest radius a and the smallest separation d so that nulls are formed in the directions $\theta = 0^\circ$, 60°, 90°, 120°, and 180°, where θ is the angle measured from the positive z axis.



6. Design a linear array of isotropic elements placed along the z axis such that the nulls of the array factor occur at $\theta = 60^\circ$, $\theta = 90^\circ$, and $\theta = 120^\circ$. Assume that the elements are spaced a distance $d = \lambda/4$ apart and that $\beta = 45^\circ$.

   a) Sketch and label the visible region on the unit circle.
   b) Find the required number of elements.
   c) Determine the excitation coefficients.

   Hint: The array factor of an N-element linear array is given by $AF = \sum_{n=1}^{N} a_n e^{\mathrm{j}(n-1)\psi}$, where $\psi = kd\cos\theta + \beta$. Use the representation $z = e^{\mathrm{j}\psi}$.