Chapter 10 Travelling Wave and Broadband Antennas

10.2 Travelling Wave Antennas

Previously, linear wire antennas were discussed and the amplitude current distribution was

1. constant for infinitesmal dipoles ($l \leq \lambda/50$)
2. linear for short dipoles ($\frac{\lambda}{50} \leq l \leq \lambda/10$)
3. sinusoidal for long dipoles ($l > \lambda/10$)

In all cases the phase distribution was assumed to be constant. The sinusoidal amplitude current distribution of long open-ended linear antennas is a standing wave. This standing wave distribution is formed by two waves of equal amplitude and 180° phase difference at the open end and travelling in opposite directions along the dipole.

Linear antennas that exhibit current (and voltage) standing wave patterns are referred to as standing wave or resonant antennas

One can also design antennas which have travelling wave patterns in current and voltage. This can be achieved by properly terminating the antenna wire so that the reflections are minimized if not compleately eliminated.

In general, all antennas whose current and voltage distribution can be represented by one or more travelling waves (as a rule usually in the same direction) are referred to as travelling wave or nonresonant antennas.

A travelling wave may be classified as a slow wave if it has a phase velocity ($V_p = \frac{\omega}{k}$) equal or smaller than the speed of light. A fast wave has a phase velocity greater than the speed of light.

An example of a slow wave travelling antenna is a long wire. An antenna is usually classified as a long wire antenna if it is a straight conductor with a length from one to several wavelengths.

As the wave travels along the wire from the source toward the load, it continuously leaks energy. Therefore the current distribution of the forward travelling wave along the structure can be represented by

$$I_f = \hat{z}\, I_z(z')\, e^{-\gamma(z')z'} = \hat{z}\, I_0\, e^{-[\alpha(z') + jk_z(z')]z'}$$

α = attenuation constant

k_z = phase constant

Simplifications:

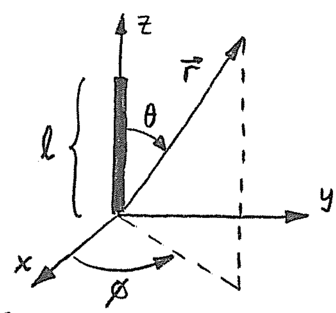
a) $\boxed{I_z(z') = I_0}$ i.e., the current is assumed constant (no amplitude modulation)

b) Ohmic losses of the wire as well as the loss of energy in a long wire ($l \gg \lambda$) due to the leakage are neglected $\Rightarrow$ $\boxed{\alpha(z') = 0}$

c) the phase constant $k_z(z')$ is assumed independent of space variables $\Rightarrow$ it corresponds to a uniform transmission line or wire as the wave travels along the structure $\Rightarrow$ $\boxed{k_z = \text{const}}$

So, the current distribution can be approximated by

$$\boxed{\vec{I} = \hat{z}\, I_0\, e^{-jk_z z'}}$$



Now, we can use techniques outlined in Chapter 4 and calculate the fields radiated by this dipole in the far-field approximation. It can be shown that these fields are given by

Not difficult!

$$E_r \approx E_\phi = H_r = H_\theta = 0$$

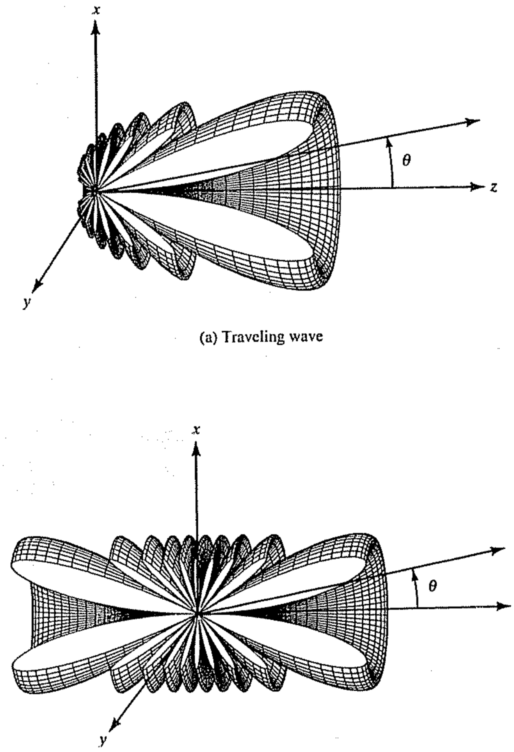
$$E_\theta \approx j\eta \frac{k I_0 l e^{-jkr}}{4\pi r}\, e^{-j\frac{kl}{2}(K - \cos\theta)} \sin\theta\, \frac{\sin\left[\frac{kl}{2}(\cos\theta - K)\right]}{\frac{kl}{2}(\cos\theta - K)} \qquad (10\text{-}2)$$

$$H_\phi \approx \frac{E_\theta}{\eta}$$

where $K = \dfrac{k}{k_z}$

From the radiated fields we can calculate the radiated power density, the total radiated power, the radiation pattern, radiation resistance, directivity, etc.

The radiation pattern shows a multilobe structure. We can compare with the pattern of a standing wave wire antenna.



(a) Traveling wave

(b) Standing wave

Figure 10.4 Three-dimensional free-space amplitude pattern for traveling and standing wave wire antennas of $l = 5\lambda$.

Travelling wave: the pattern is formed by a forward travelling wave current $I_1 e^{-jkz}$

Standing wave: the pattern is formed by forward plus backward travelling wave currents

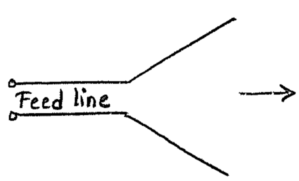
$$I_1 e^{-jkz} - I_2 e^{jkz} = -2jI_0 \sin(kz)$$

(when $I_1 = I_2 = I_0$)

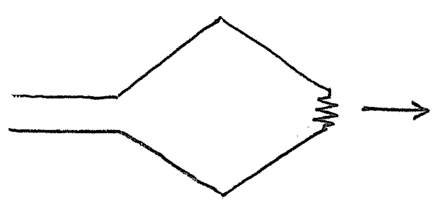
From the Figure it is seen that the travelling wave antenna is used when it is desired to radiate or receive predominantly from one direction.

As the length of the wire increases, the maximum of the main lobe shift closer toward the axis and the number of lobes increases.

10.2.2 V Antenna



10.2.3 Rhombic Antenna



10.3 Broadband Antennas

10.3.1 Helical Antenna

Another basic, simple and practical antenna is that of a conducting wire wound in the form of a screw thread forming a helix.

The geometrical configuration of a helix consists usually of N turns, diameter D and spacing S between each turn. The total length of the antenna is $L = NS$, while the total length of the wire is $L_n = NL_0 = N\sqrt{S^2+C^2}$, where L_0 is the length of the wire between each turn, and $C = \pi D$ is the circumference of the helix.

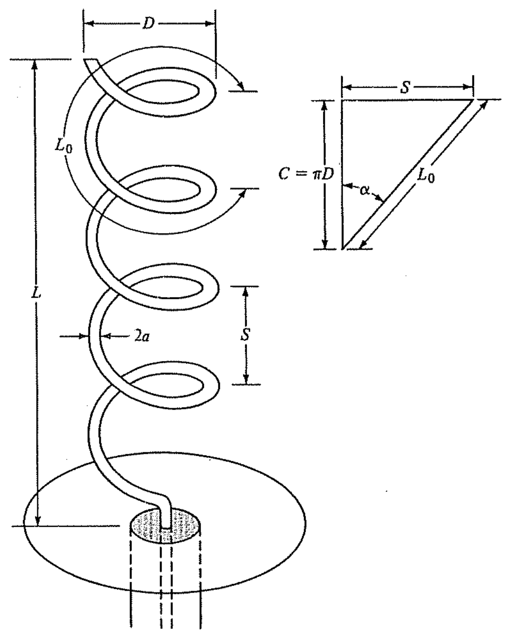


Figure 10.13 Helical antenna with ground plane.

An important parameter for the helical antenna is the pitch angle α which is the angle formed by a line tangent to the helix wire and a plane perpendicular to the helix axis

$$\boxed{\alpha = \arctan\left(\frac{S}{\pi D}\right) = \arctan\left(\frac{S}{C}\right)}$$

When $\alpha = 0°$ the winding is flattened and the helix reduces to a loop antenna of N turns. If $\alpha = 90°$ the helix reduces to a linear wire. When $0° < \alpha < 90°$ a true helix is formed.

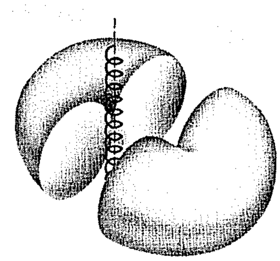
The radiation characteristics of the antenna can be varied by controlling the size of its geometrical properties compared to the wavelength. The helical antenna can operate in many modes. However there are two principal modes of operation:

1) the normal mode (broadside)
2) the axial mode (end-fire)

The axial mode is most efficient and most practical because it can achieve circular polarization over a wider bandwidth.

Normal (broadside) mode

In this mode of operation the radiated field is maximum in a plane normal to the helix axis and minimum along its axis. The radiation pattern is similar to that of a linear dipole of $l < \lambda$, or a small loop ($a \ll \lambda$) (so called omnidirectional pattern).



To achieve the normal mode of operation, the dimensions of the helix are usually small compared to the wavelength, i.e., $NL_0 \ll \lambda$

Since the limiting geometries of the helix are loops of diameter D ($\alpha \to 0°$) and a linear wire ($\alpha \to 90°$), the far-field radiated by a small helix in the normal mode can be described in terms of E_θ (dipole) and E_ϕ (loop) components. So, we can consider the helical antenna operating in normal mode as consisting of N small loops and N short dipoles connected together in a series.

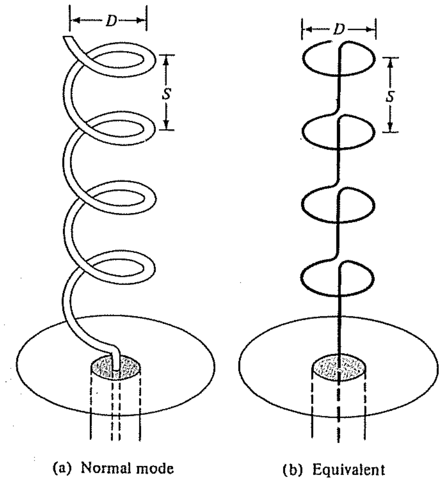


Figure 10.14 Normal (broadside) mode for helical antenna and its equivalent.

Since in normal mode operation the helix dimensions are small, the current through its length can be assumed to be constant and the total radiated field can be described by the sum of the fields radiated by a small loop of radius D (E_ϕ) and a short dipole of length S (E_θ)

$$\vec{E} = \hat{\theta} E_\theta + \hat{\phi} E_\phi$$

where

$$E_\theta = j\eta \frac{kI_0 S e^{-jkr}}{4\pi r} \sin\theta \quad \text{and} \quad E_\phi = \eta \frac{k(D/2)^2 I_0 e^{-jkr}}{4r} \sin\theta$$

The ratio of the magnitudes is defined as the axial ratio

$$\boxed{AR = \frac{|E_\theta|}{|E_\phi|} = \frac{2\lambda S}{(\pi D)^2}}$$

The most interesting from the practical point of view is the case when $AR = 1$, i.e.,

$$\frac{2\lambda S}{(\pi D)^2} = 1 \Rightarrow C = \pi D = \sqrt{2\lambda S} \qquad \left(\text{or } S = \frac{(\pi D)^2}{2\lambda}\right)$$

$$\tan\alpha = \frac{S}{\pi D} = \frac{(\pi D)^2}{2\lambda \pi D} = \frac{\pi D}{2\lambda}$$

When the dimensional parameters of the helix satisfy these relationships, the radiated field is circularly polarized in all directions other than $\theta = 0°$ where the field vanish.

Conclusion: because of the critical dependence of its radiation characteristics on its geometrical dimensions, which must be very small compared to the wavelength, this mode of operation is very narrow in bandwidth and its radiation efficiency is very small. This mode of operation is seldom utilized.

Axial (end-fire) mode

This is a more practical mode of operation. In this mode there is only one major lobe and its maximum radiation intensity is along the axis of the helix. To excite this mode, the diameter D and the spacing S must be large fractions of λ.

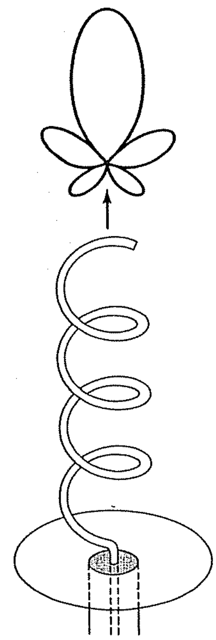


Figure 10.15 Axial (endfire) mode of helix.

In order to obtain circular polarization, primarily in the main lobe, we must have

$$\begin{cases} \frac{3}{4} < \frac{C}{\lambda} < \frac{4}{3} & \left(\text{with } \frac{C}{\lambda} = 1 \text{ near optimum}\right) \\ S \approx \lambda/4 \\ 12^\circ < \alpha < 14^\circ \end{cases}$$

Usually the antenna is used in conjunction with a ground plane and is fed by a coaxial line.

Input impedance $R \approx 140\left(\frac{C}{\lambda}\right) \quad \pm 20\%$ Values: $100\,\Omega - 200\,\Omega$

$$\text{HPBW (degrees)} \approx \frac{52\lambda^{3/2}}{C\sqrt{NS}}$$

$$\text{Directivity} \quad D_0 \approx 15N\frac{C^2 S}{\lambda^3}$$

$$\text{Axial ratio} \quad AR = \frac{2N+1}{2N}$$

To obtain the far-field pattern one assumes that the helix consists of an array of N identical turns, a uniform spacing between them and that the elements are placed along the z-axis. The total radiated field is obtained by multiplying the field from one turn with the array factor = pattern multiplication.

Normalized far-field pattern:

$$E = \sin\left(\frac{\pi}{2N}\right)\cos\theta \frac{\sin\left[\frac{N}{2}\psi\right]}{\sin\left[\frac{\psi}{2}\right]}$$

$$\psi = k_0\left(S\cos\theta - \frac{L_0}{p}\right) \quad \text{where } p = \begin{cases} \dfrac{L_0/\lambda_0}{S/\lambda_0 + 1} & \text{ordinary end-fire} \\ \dfrac{L_0/\lambda}{\frac{S}{\lambda_0} + \left(\frac{2N+1}{2N}\right)} & \text{Hansen-Woodyard end-fire} \end{cases}$$

10.3.3 Yagi-Uda Array of Linear Elements

This is a very practical radiator in the frequency bands

- HF (3-30 MHz)
- VHF (30-300 MHz)
- UHF (300-3,000 MHz)

At those frequencies, these antennas assume handy dimensions.

The are, for example, used as TV-antennas.

One element is fed/driven and the other are parasitic. A reflector is put behind the driven element and directors in the direction of desired radiation (along the y-axis in the Figure). The directivity increases with the number of directors. A second reflector may also be used, but that will not increase the performance by any significant amount. The feed element is commonly a folded dipole with impedance 300Ω, see Figure 9.12.

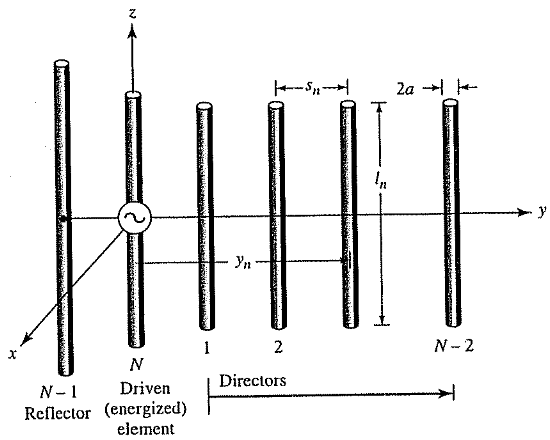


Figure 10.17 Yagi-Uda antenna configuration.

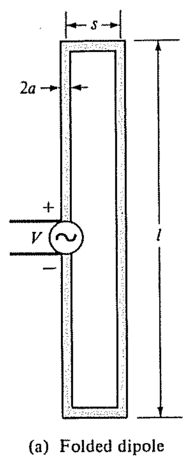


Figure 9.12 Folded dipole and

Lengths of elements*

Driven:	$(0.45 - 0.49)\lambda$	
Directors:	$(0.4 - 0.45)\lambda$	but not necessarily of the same length or diameter
Reflector:	Sometimes longer than the driven element	

Separation between elements

Director-director: $(0.3-0.4)\lambda$, but not necessarily uniform

Driven element-reflector: a bit smaller than the spacing between the driven element and the nearest director and is about 0.25λ for optimum.

The total phase of the current induced in the directors depend upon their lengths and their spacing to the adjacent elements.

The number of directors is usually between 6 and 12.
More directors ⇒ higher directivity, but there is a limit beyond which very little is gained by adding more directors.

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* These lengths are for the first ($\lambda/2$) resonance. Yagi-Uda antennas can also be designed for higher resonances (λ, $3\lambda/2$, ...), but seldom are.

Narrow bandwidth ~2 %

Numerical techniques are often used to optimize the performance, which depends on three parts:

1. the reflector-feeder arrangement
2. the feeder
3. the rows of directors

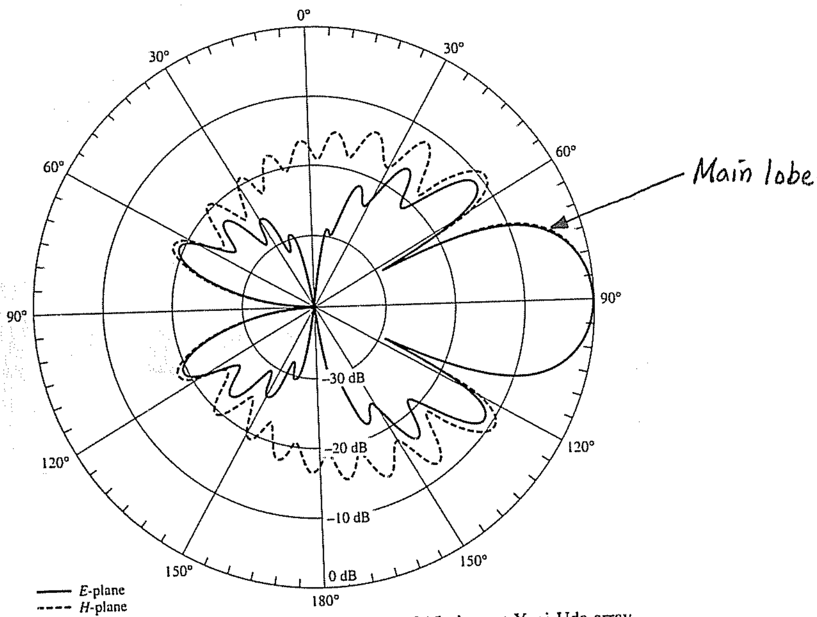


Figure 10.19 *E*- and *H*-plane amplitude patterns of 15-element Yagi-Uda array.

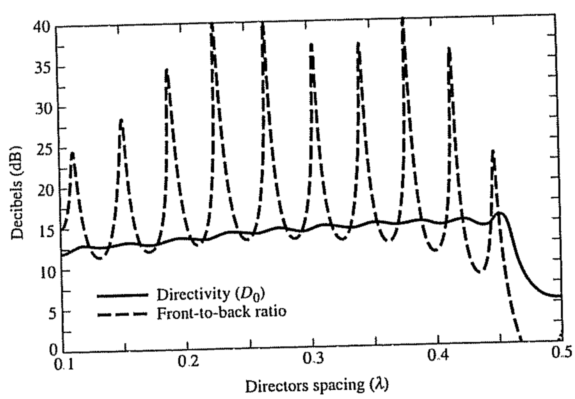
Main lobe: controlled with the directors

Back lobes: controlled with the reflector spacing and size, the director spacing and size, and the feeder length and radius.

Input impedance: depends on the reflector spacing, the feed length and radius, and the director spacing and size.

Gain: depends largely on the number of directors and their spacing and size. Does not depend much on the reflector and the feeder length and size.

Use the computer program YAGI-UDA ARRAY to check



The front-to-back lobe ratio is highly dependent on the director spacing.

Figure 10.22 Directivity and front-to-back ratio, as a function of director spacing, for 15-element Yagi-Uda array. of Figure 10.19

After the design, the Yagi-Uda array can be optimized further by, for example
- allowing for nonuniform spacing between the directors while keeping the lengths fixed.
- allowing for nonuniform lengths while keeping the spacing fixed
- allowing for nonuniform spacing and lengths simultaneously

Design Procedure

Design by using
1) Table 10.6 which represents optimized antenna parameters for six different lengths and for $d/\lambda = 0.0085$
2) Figure 10.25 which represents uncompensated director and reflector lengths for $0.001 \leq d/\lambda \leq 0.04$
3) Figure 10.26 which provides compensation length increase for all the parasitic elements (directors & reflectors) as a function of boom-to-wavelength ratio $0.001 \leq D/\lambda \leq 0.04$

Using 1)-3) we can, according to the Table 10.6, only design arrays of lengths 0.4, 0.8, 1.2, 2.2, 3.2, and 4.2 times the wavelength.

Specify the center frequency, directivity, d/λ, and D/λ and find optimum parasitic element lengths (directors & reflector). However, the spacing between the reflector and the driven element is $s = 0.2\lambda$ for all designs.

Table 10.6 OPTIMIZED UNCOMPENSATED LENGTHS OF PARASITIC ELEMENTS FOR YAGI-UDA ANTENNAS OF SIX DIFFERENT LENGTHS

$d/\lambda = 0.0085$, $s_{12} = 0.2\lambda$		LENGTH OF YAGI-UDA (IN WAVELENGTHS)					
		0.4	0.8	1.20	2.2	3.2	4.2
LENGTH OF REFLECTOR (l_1/λ)		0.482	0.482	0.482	0.482	0.482	0.475
LENGTH OF DIRECTORS, λ	l_3	0.442	0.428	0.428	0.432	0.428	0.424
	l_4		0.424	0.420	0.415	0.420	0.424
	l_5		0.428	0.420	0.407	0.407	0.420
	l_6			0.428	0.398	0.398	0.407
	l_7				0.390	0.394	0.403
	l_8				0.390	0.390	0.398
	l_9				0.390	0.386	0.394
	l_{10}				0.390	0.386	0.390
	l_{11}				0.398	0.386	0.390
	l_{12}				0.407	0.386	0.390
	l_{13}					0.386	0.390
	l_{14}					0.386	0.390
	l_{15}					0.386	0.390
	l_{16}					0.386	
	l_{17}					0.386	
SPACING BETWEEN DIRECTORS (s_{ik}/λ)		0.20	0.20	0.25	0.20	0.20	0.308
DIRECTIVITY RELATIVE TO HALF-WAVE DIPOLE (dB)		7.1	9.2	10.2	12.25	13.4	14.2
DESIGN CURVE (SEE FIGURE 10.25)		(A)	(B)	(B)	(C)	(B)	(D)

SOURCE: Peter P. Viezbicke, *Yagi Antenna Design*, NBS Technical Note 688, December 1976.

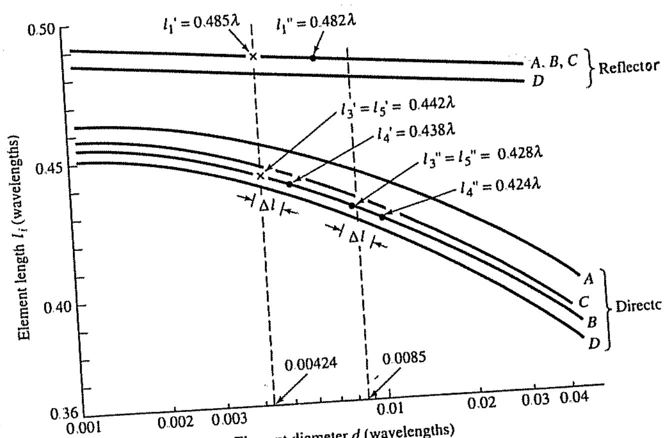


Figure 10.25 Design curves to determine element lengths of Yagi-Uda arrays. (SOURCE: P. P. Viezbicke, "Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

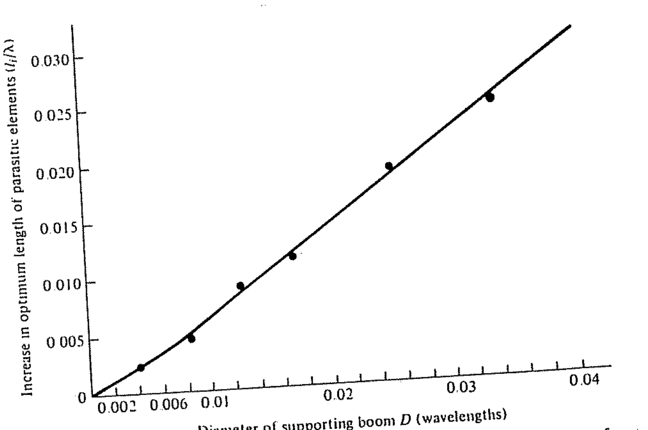


Figure 10.26 Increase in optimum length of parasitic elements as a function of metal boom diameter. (SOURCE: P. P. Viezbicke, "Yagi Antenna Design," NBS Technical Note 688, U.S. Department of Commerce/National Bureau of Standards, December 1976)

Example 10.2

Design a Yagi-Uda array with a directivity (relative to a $\lambda/2$ dipole at the same height above ground) of 9.2 dB at $f_0 = 50.1$ MHz. The desired diameter of the parasitic elements is 2.54 cm and of the metal supporting boom 5.1 cm. Find the element spacings, lengths, and total array length.

SOLUTION

(a) At $f_0 = 50.1$ MHz the wavelength is $\lambda = 5.988$ m $= 598.8$ cm. Thus $d/\lambda = 2.54/598.8 = 4.24 \times 10^{-3}$ and $D/\lambda = 5.1/598.8 = 8.52 \times 10^{-3}$.

(b) From Table 10.6, the desired array would have a total of five elements (three directors, one reflector, one feeder). For a $d/\lambda = 0.0085$ ratio the optimum uncompensated lengths would be those shown in the second column of Table 10.6 ($l_3 = l_5 = 0.428\lambda$, $\lambda_4 = 0.424\lambda$, and $l_1 = 0.482\lambda$). The overall antenna length would be $L = (0.6 + 0.2)\lambda = 0.8\lambda$, the spacing between directors 0.2λ, and the reflector spacing 0.2λ. It is now desired to find the optimum lengths of the parasitic elements for a $d/\lambda = 0.00424$.

(c) Plot the optimized lengths from Table 10.6 ($l_3'' = l_5'' = 0.428\lambda$, $l_4'' = 0.424\lambda$, and $l_1'' = 0.482\lambda$) on Figure 10.25 and mark them by a dot ($\cdot$).

(d) In Figure 10.25 draw a vertical line through $d/\lambda = 0.00424$ intersecting curves (*B*) at director uncompensated lengths $l_3' = l_5' = 0.442\lambda$ and reflector length $l_1' = 0.485\lambda$. Mark these points by an x.

(e) With a divider, measure the distance (Δl) along director curve (*B*) between points $l_3'' = l_5'' = 0.428\lambda$ and $l_4'' = 0.424\lambda$. Transpose this distance from the point $l_3' = l_5' = 0.442\lambda$ on curve (*B*), established in step (d) and marked by an x, downward along the curve and determine the uncompensated length $l_4' = 0.438\lambda$. Thus the boom uncompensated lengths of the array at $f_0 = 50.1$ MHz are

$$l_3' = l_5' = 0.442\lambda$$
$$l_4' = 0.438\lambda$$
$$l_1' = 0.485\lambda$$

(f) Correct the element lengths to compensate for the boom diameter. From Figure 10.26, a boom diameter-to-wavelength ratio of 0.00852 requires a fractional length increase in each element of about 0.005λ. Thus the final lengths of the elements should be

$$l_3 = l_5 = (0.442 + 0.005)\lambda = 0.447\lambda$$
$$l_4 = (0.438 + 0.005)\lambda = 0.443\lambda$$
$$l_1 = (0.485 + 0.005)\lambda = 0.490\lambda$$

10.3.4 Yagi-Uda Array of Loops

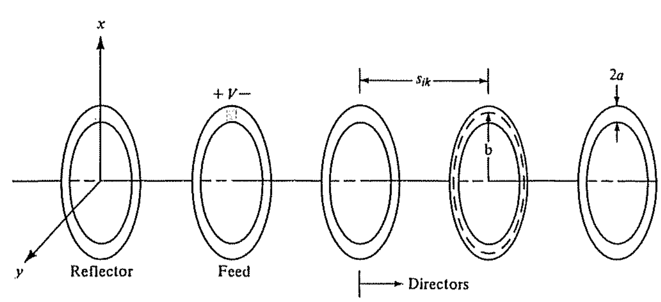


Figure 10.27 Yagi-Uda array of circular loops.