The Effect of Irregularities on the Direct Current

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History

- not refereed: Buchert, S. and S. Saito, On the Pedersen current which is carried by electrons, in *Substorms-4*, edited by S. Kokubun and Y. Kamide, Terra Scientific Publishing, Tokyo, 1998
- manuscript by Saito and Buchert, rejected by GRL
- manuscript by Hagfors et al., rejected by Ann. Geophys.
- `Manuscript Number: 2004JA010788RRRRR`
  `Manuscript Title: Effect of Electrojet Irregularities on DC Current Flow`

  `Dear Stephan:`

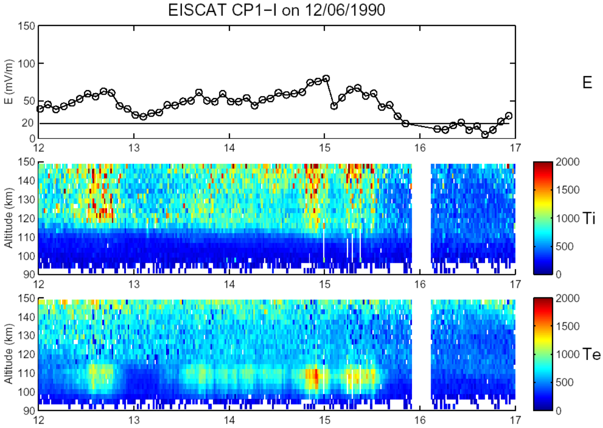
  `I am pleased to accept the above manuscript for publication in the Journal of Geophysical ...`
- JGR, 2006:
  `http://www.agu.org/journals/ja/ja0602/2004JA010788/`

Special thanks to A. Richmond, Editor of JGR

Electrojet Irregularities

- occur in the E-region ionosphere at altitudes $\approx$ 90–120 km,
- at the magnetic equator, in the auroral zone, sometimes at mid latitudes
- mainly field-aligned density variations seen by radars as Bragg scattering
- typical wavelengths 1–30 m
- explained by the Farley-Buneman instability
  $\rightarrow$ ion and e^- velocity difference exceeds ion sound velocity
- electric current mainly a Hall current?

Heating of the Ionosphere



IS radar (EISCAT) measures electric field $\mathbf{E}_0$, T_e, and T_i. Whenever $|\mathbf{E}_0| >$ a threshold ca 30 mV/m, then

- Electron heating at altitudes $\approx$ 98–115 km
- T_e increases from $\approx$ 300 K up to $\approx$ 2000 K
- Ion heating at altitudes above $\approx$ 120 km

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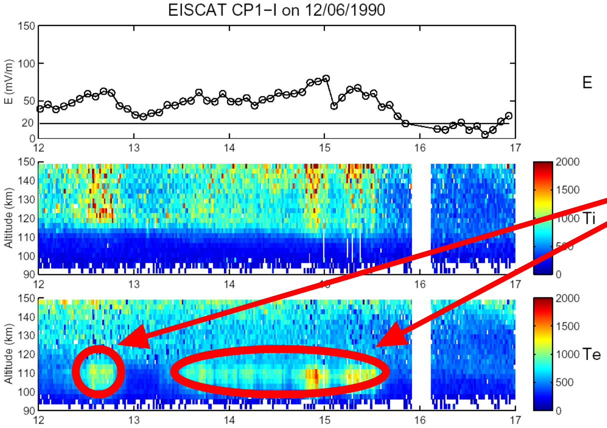

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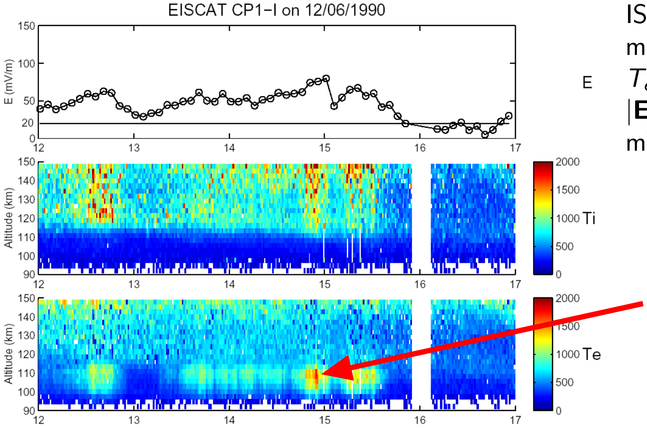

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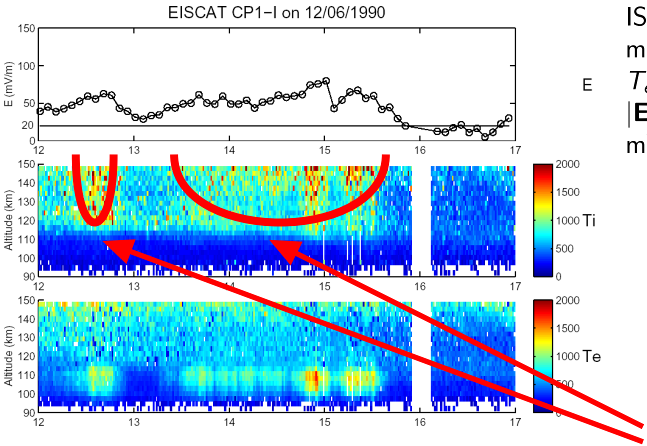

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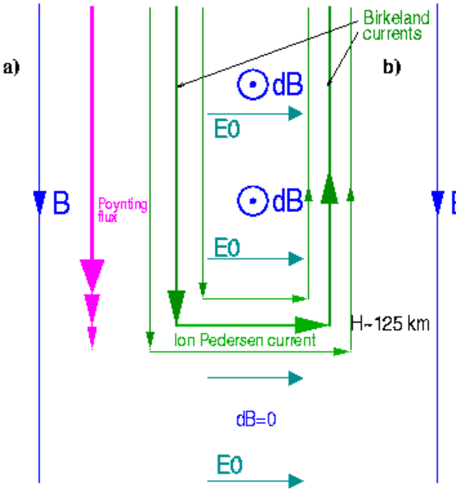
Collisional Ion Heating



- Ion heating due to ion-neutral collisions and imposed $\mathbf{E}_0$
- Ion-neutral collisions demagnetize ions, $45^\circ\angle$ between ion drift and ExB at altitude $\approx$ 130 km
- dissipative Pedersen current $\mathbf{j}_P$ closes Birkeland current
- heating rate $\mathbf{j}_P \cdot \mathbf{E}_0$
- magnetic effect of currents equivalent to convergence of Poynting flux $\mathbf{S}$, $\nabla \cdot S = -\mathbf{j}_P \cdot \mathbf{E}_0$
- transfer of electromagnetic energy from (far) above into the polar ionosphere
- ultimately the neutral upper atmosphere is heated

How about the Electron Heating?

- e^- collision frequency $\nu_e \ll \Omega_e$ e^- gyrofrequency

$$\mathbf{E_0} = -\frac{m}{e} \left\{ \begin{matrix} \nu_e & -\Omega_e & 0 \\ +\Omega_e & \nu_e & 0 \\ 0 & 0 & \nu_e \end{matrix} \right\} \mathbf{v_0} \qquad (1)$$

- zero order e^- drift $\mathbf{v}_0 \approx \mathbf{E_0} \times \mathbf{B}/B^2$

1998: We (don't need no ... theory and) postulate that in the presence of irregularities

- the mean electron drift $\langle \mathbf{v} \rangle \neq \mathbf{v}_0$
- the mean current $\langle \mathbf{j} \rangle$ is partially a Pedersen current, $\langle \mathbf{j} \rangle \cdot \mathbf{E_0} > 0$ even in the lower E region

Plan: parameterize the effective $\sigma_P^*(|\mathbf{E_0}|)$ using EISCAT data, to improve conductivity models for AMIE ...

The Plan

- assume that a density spectrum $\left\langle \left| N_1 \left(\mathbf{k}, \omega \right) \right|^2 \right\rangle$ is given (by theory, simulation ...) or has been measured
- calculate the mean current $\langle \mathbf{j} \rangle$ for this density spectrum and then the external (magnetospheric) power input $\langle \mathbf{j} \rangle \cdot \mathbf{E}_0$
- calculate also the mean Joule heating rate $\langle \mathbf{j} \cdot \mathbf{E} \rangle$ (wave heating?)

Zero and first order quantities

Current

$$\mathbf{j}(\mathbf{r},t) = e\left(N(\mathbf{r},t)\,\mathbf{V}(\mathbf{r},t) - n(\mathbf{r},t)\,\mathbf{v}(\mathbf{r},t)\right) \tag{2}$$

$N(\mathbf{r},t)$ and $\mathbf{V}(\mathbf{r},t)$ ion density and velocity
$n(\mathbf{r},t)$ and $\mathbf{v}(\mathbf{r},t)$ e^- density and velocity

$$\begin{aligned} N(\mathbf{r},t) &= N_0 + N_1(\mathbf{r},t) \\ \mathbf{V}(\mathbf{r},t) &= \mathbf{V}_0 + \mathbf{V}_1(\mathbf{r},t) \\ n(\mathbf{r},t) &= n_0 + n_1(\mathbf{r},t) \\ \mathbf{v}(\mathbf{r},t) &= \mathbf{v}_0 + \mathbf{v}_1(\mathbf{r},t)\,. \end{aligned}$$

Mean quantities

$$\langle f(\mathbf{r},t)\rangle = \frac{1}{V}\int_V d(\mathbf{r})\,\frac{1}{T}\int_T dt f(\mathbf{r},t)\,.$$

The mean current

$$\langle \mathbf{j}(\mathbf{r},t)\rangle = e(N_0\mathbf{V}_0 - n_0\mathbf{v}_0 + \langle N_1(\mathbf{r},t)\,\mathbf{V}_1(\mathbf{r},t)\rangle - \langle n_1(\mathbf{r},t)\,\mathbf{v}_1(\mathbf{r},t)\rangle) \qquad (3)$$

is affected by correlations between densities and velocities.

Fourier transform

First order ion current

$$\langle \mathbf{j}_{i1}(\mathbf{r},t)\rangle = \left(\frac{1}{2\pi}\right)^4 \frac{e}{VT} \iiiint d(\mathbf{k}) d\omega \left\langle \Re\left(\mathbf{V}_1(\mathbf{k},\omega) N_1^*(\mathbf{k},\omega)\right)\right\rangle \quad (4)$$

and similarly for the e$^-$ current

$$\langle \mathbf{j}_{e1}(\mathbf{r},t)\rangle = -\left(\frac{1}{2\pi}\right)^4 \frac{e}{VT} \iiiint d(\mathbf{k}) d\omega \left\langle \Re\left(\mathbf{v}_1(\mathbf{k},\omega) n_1^*(\mathbf{k},\omega)\right)\right\rangle \quad (5)$$

Next establish relation between first order velocities and densities.

Continuity and Momentum Equations

$$
\begin{aligned}
0 &= -i\omega N_1(\mathbf{k},\omega) + i\mathbf{k}\cdot\mathbf{V}_1(\mathbf{k},\omega) N_0 \\
0 &= -in_1(\mathbf{k},\omega)(\omega - \mathbf{k}\cdot\mathbf{v}_0) + i\mathbf{k}\cdot\mathbf{v}_1(\mathbf{k},\omega) n_0 \\
-i\omega\mathbf{V}_1(\mathbf{k},\omega) &= \frac{e}{M}\mathbf{E}_1(\mathbf{k},\omega) - i\mathbf{k}\frac{\kappa T_i}{M}\frac{N_1(\mathbf{k},\omega)}{N_0} - \nu_i\mathbf{V}_1(\mathbf{k},\omega) \\
\frac{e}{m}(\mathbf{v}_1(\mathbf{k},\omega)\times\mathbf{B}_0) &= -\frac{e}{m}\mathbf{E}_1(\mathbf{k},\omega) - i\mathbf{k}\frac{\kappa T_e}{m}\frac{n_1(\mathbf{k},\omega)}{n_0} - \nu_e\mathbf{v}_1(\mathbf{k},\omega)
\end{aligned}
\tag{6}
$$

Assumptions

- zero e^- mass
- no effect of the magnetic field on the ions, $\mathbf{V}_0 = 0$
- quasi-neutrality, $n_0 = N_0$ and $n_1 = N_1$
- $\mathbf{k}$ component parallel to B negligible
- imaginary part of ω small compared to real part

Dispersion Relation for Farley-Buneman instability

$$(\omega - \mathbf{k}\cdot\mathbf{v}_0) = \frac{M}{m}\left(\frac{\omega(i\omega - \nu_i)}{k^2} - iC_s^2\right)\left(\frac{\nu_e(k_x^2 + k_y^2)}{\Omega_e^2 + \nu_e^2} + \frac{k_z^2}{\nu_e}\right) \qquad (7)$$

$$\omega_r = \frac{\mathbf{k}\cdot\mathbf{v}_0}{1 + \Psi_0} \qquad (8)$$

where Ψ_0 has the usual meaning:

$$\Psi_0 = \frac{M}{m}\frac{\nu_i}{k^2}\left(\frac{\nu_e(k_x^2 + k_y^2)}{\Omega_e^2 + \nu_e^2} + \frac{k_z^2}{\nu_e}\right) \approx \frac{\nu_e\nu_i}{\Omega_e\Omega_i} \qquad (9)$$

Mean Current

$$\langle \mathbf{j}(\mathbf{r},t)\rangle = - eN_0\mathbf{v}_0 + \left(\frac{1}{2\pi}\right)^3 \frac{e}{VT} \iiint d(\mathbf{k})\mathbf{A}\frac{\mathbf{k}\cdot\mathbf{v}_0}{1+\Psi_0}\frac{\langle |N_1(\mathbf{k},\omega_r)|^2\rangle}{N_0} \quad (10)$$

Vector **A** has components:

$$\begin{aligned} A_x &= \frac{k_x}{k^2} + \frac{M}{m}\left(\frac{\nu_i}{k^2}\right)\frac{k_x\nu_e - k_y\Omega_e}{\Omega_e^2+\nu_e^2} \\ A_y &= \frac{k_y}{k^2} + \frac{M}{m}\left(\frac{\nu_i}{k^2}\right)\frac{k_y\nu_e + k_x\Omega_e}{\Omega_e^2+\nu_e^2} \\ A_z &= \frac{k_z}{k^2} + \frac{M}{m}\left(\frac{\nu_i}{k^2}\right)\frac{k_z}{\nu_e} \end{aligned} \quad (11)$$

External Power Input

$$\langle \mathbf{j} \rangle \cdot \mathbf{E}_0 \approx \frac{1}{VT} \left(\frac{1}{2\pi} \right)^3 M\nu_i \iiint d(\mathbf{k}) \frac{(\mathbf{k} \cdot \mathbf{v}_0)^2}{k^2(1 + \Psi_0)} \frac{\left\langle |N_1(\mathbf{k}, \omega_r)|^2 \right\rangle}{N_0} \tag{12}$$

Mean Joule Heating Rate

$$\langle \mathbf{j} \cdot \mathbf{E} \rangle = \langle \mathbf{j} \rangle \cdot \mathbf{E}_0 + \langle \mathbf{E_1}(\mathbf{r}, t) \cdot \mathbf{j_1}(\mathbf{r}, t) \rangle \tag{13}$$

Split $\mathbf{E_1}(\mathbf{r}, t) \cdot \mathbf{j_1}(\mathbf{r}, t)$ into

$$\begin{aligned} I_1 &= e\mathbf{E_1}(\mathbf{r}, t) \cdot (N_1(\mathbf{r}, t)\mathbf{V_0} - n_1(\mathbf{r}, t)\mathbf{v_0}) \\ I_2 &= e\mathbf{E_1}(\mathbf{r}, t) \cdot (\mathbf{V_1}(\mathbf{r}, t)N_0 - \mathbf{v_1}(\mathbf{r}, t)n_0) \end{aligned} \tag{14}$$

I_1 affected by correlations between electric field and densities, I_2 by correlations between electric field and velocities.

Fourier Transform and Averaging

$$L_1 = -\left(\frac{1}{2\pi}\right)^3 \frac{M\nu_i}{VT} \int d(\mathbf{k}) \frac{(\mathbf{k}\cdot\mathbf{v_0})^2}{k^2(1+\Psi_0)} \frac{\left\langle |N_1(\mathbf{k},\omega_r)|^2 \right\rangle}{N_0} = -\langle \mathbf{j} \rangle \cdot \mathbf{E}_0 \quad (15)$$

and

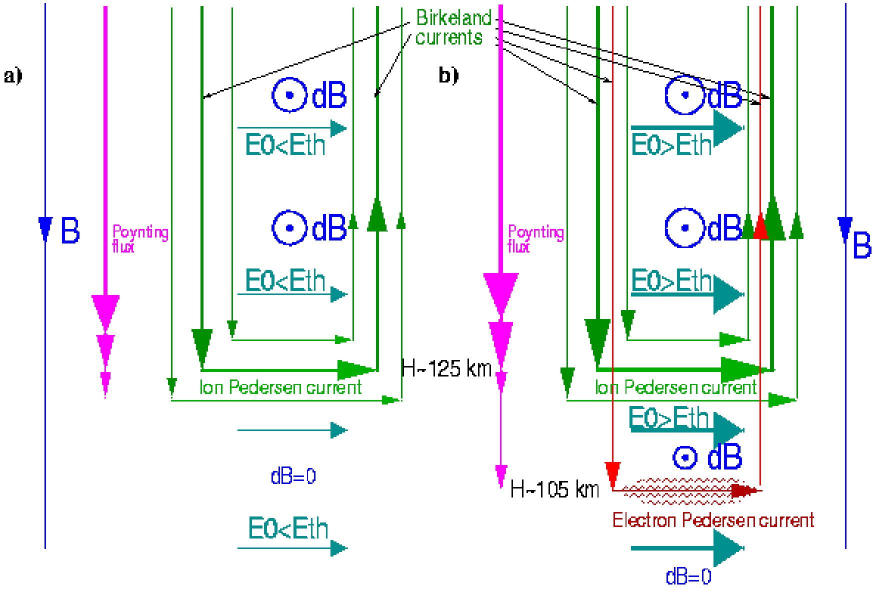
$$L_2 = \left(\frac{1}{2\pi}\right)^3 \frac{M\nu_i}{VT} \int d(\mathbf{k}) \frac{(\mathbf{k}\cdot\mathbf{v_0})^2}{k^2(1+\Psi_0)} \frac{\left\langle |N_1(\mathbf{k},\omega_r)|^2 \right\rangle}{N_0} = +\langle \mathbf{j} \rangle \cdot \mathbf{E}_0 \quad (16)$$

$$L_1 + L_2 = 0$$

Wave Heating?

- Average wave heating $\langle \mathbf{j}_1 \cdot \mathbf{E}_1 \rangle = 0!$
- External power input $\langle \mathbf{j} \rangle \cdot \mathbf{E}_0 = \langle \mathbf{j} \cdot \mathbf{E} \rangle$ mean Joule heating
- irregularities affect the DC current, and this alone accounts for the e^- heating

Summary



Birkeland currents
a)
b)
dB
E0<Eth
E0>Eth
B
Poynting flux
H~125 km
Ion Pedersen current
H~105 km
Electron Pedersen current
dB=0

Conclusions

- Irregularities affect the perpendicular DC
- The ionospheric Pedersen conductivity is effectively non-linear, it depends on the electric field
- Plasma is transported anomalously along $\mathbf{E}_0$, eg from the bright to the black aurora (this might explain why auroral arcs can exist a long time)
- The velocity difference between ions and e^- is the microphysical cause of the FB instability,
- but the free energy for maintaining a stationary turbulent state is external electromagnetic energy.
- there is no “wave heating” in irregularities, $\langle \mathbf{j}_1 \cdot \mathbf{E}_1 \rangle = 0$

Questions/Outlook

- complete the original plan, $\sigma_P^*(|\mathbf{E}_0|?$, using data
- can a corresponding generator be found, for example at the magnetopause?
  - experimentally, with Cluster data?
  - theoretically, eg with lower hybrid waves/irregularities/turbulence
- parallel to $\mathbf{B}_0$ waves/irregularities don't affect the DC (to first order), rather a quasi-stationary $E_{||}$ is set up
- theoretical prove that this is actually occuring?
- like closure of $j_{||}$ also $E_{||}$ causes a divergence of the downward Poynting flux, and this powers the aurora!
- (the velocity difference between ions and electrons due to $j_{||}$ provides free energy for certain microinstabilities, but it does not provide any significant energy to the aurora)