Cluster EFW, Introduction to Calibrations

Georg Gustafsson
Swedish Institute of Space Physics, Uppsala Division
S-755 91 Uppsala, Sweden

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1 Introduction

1.1 Background

The electric field and wave experiment (EFW) of the Cluster project is designed to measure the electric field and plasma density.

The instrument measures the wave and quasi-static electric fields in the spin plane of the four Cluster spacecraft with high time resolution. Voltage/current sweeps can also be made to measure both electron temperature and density. The three magnetic field signals from the search coil sensors are also available in the EFW experiment.

The sensors consist of 16 spherical probes, four for each of the four spacecraft. The probes can be operated in pairs to measure the voltage between probes or the voltage between a single probe and the spacecraft can be measureed. The probes can also be used as low impedance probes, Ampere meter, to measure the current between the plasma and the probe.

1.2 Scope of the Document

This document provides the background and overview of the EFW instrument calibrations.

1.3 Related Documents

1. The scientific user requirements on the EFW instrument are described in [Ref. **?**], part 1, page 8, *Scientific capabilities.*
2. The analog calibrations are described in [Ref. **?**].
3. The *digital calibrations* reference document is [Ref. **?**].
4. The application of the calibrations is described in [Ref. **?**].
5. The comparisons between analog and digital calibrations are described in [Ref. **?**].
6. Some timing issues are discussed in [Ref. **?**].
7. The mecanical mounting of units is documented in [Ref. **?**].

1.4 Definitions and Acronyms

The acronyms and abbrieviations are listed in Table 1.

Acronym	Meaning
AC	Alternating Current
DC	Direct Current
EFW	Electric Field and Waves
IRF-U	Institutet för Rymdfysik, Uppsalaavdelningen
PI	Principal Investigator
TBD	To Be Defined
TBW	To Be Written
WEC	Wave Experiment Consortium

Table 1: Acronyms and abbrieviations

2 Background to EFW Instrument Calibrations

2.1 Signal processing systems

The approach adopted here is to consider a signal processing device simply as one which delivers to its output a modified version of a signal applied to its input. In other words it is regarded as an "input-output" device, whose detailed internal construction does not concern us. It involves a mathematical or numerical description of the overall performance of the signal processor, but not the detailed electrical circuit of which it is constructed. With knowledge of the characteristics of the processor it is possible to predict the effect of a signal processing device or operation on either the waveform or on the frequency spectrum of a particular signal. Another important type of problem arises when a signal is mixed with an unwanted disturbance, and the signal is to be processed so as to reduce that disturbance as much as possible. And finally, it is often valuable to observe the signal at two or more points in a processing system and, by comparing them, to deduce something about the system's characteristics. The more common processes in a linear system are amplification/attenuation and filstering/mixing.

2.2 Linear processing systems

Linear systems form an important class of processing systems.

such systems obey the principle of superposition, and have the important property of "frequency preservation": however different the input and output waveforms of a linear system may appear, the latter never contains frequencies which are absent from the input. Such systems may be described by so-called linear differential equations with constant coefficients. Great care must, however, be exercised in the use of the word linear, because the instantaneous values of the input and output signals of a linear system are not in most cases linearly related. The principle of superposition is very important. It implies that we may consider any signal input to be made up from a number of separate components - such as sine and cosine waves - and evaluate the response to each component. The output is then found by summation of the individual responses. Thus in a linear system we can define the way in which different frequencies are modified in passing through the system, we have a powerful method of defining its response to any signal waveform.

2.2.1 The frequency-domain approach

Suppose a continuous sinusoidal signal $A\sin(\omega_0 t - \varphi)$ forms the input to a linear processing device. The output will be a wave at the same frequency, but with modified amplitude and phase. In the general case the input signal may be considered to be made up from a large number of frequency components, and the modifications to amplitude and phase caused by the system will be frequency-dependent. Let the spectrum of the input signal be denoted by $G_1(j\omega)$ (which defines both its magnitude and phase characteristics) and let the modifications to this spectrum caused by the system be denoted by the complex function $H(j\omega)$. At any value of ω, $H(j\omega)$ also has both magnitude and phase terms; the former represents a multiplication factor and the latter an imposed shift. As an example assume a linear system that magnifies a wave of frequency ω_0 by a factor B and imposes a phase lag of φ radians hence

$$H(j\omega)_{\omega=\omega_0} = H(j\omega_0) = Be^{-j\varphi} = B\cos\varphi - jB\sin\varphi \tag{1}$$

and

$$|H(j\omega_0)| = \sqrt{(B^2\cos\varphi^2 + B^2\sin\varphi^2} \tag{2}$$

Furthermore, if we denote the output signal by $G_2(j\omega)$ then

$$G_2(j\omega) = G_1(J\omega)\cdot H(j\omega) \tag{3}$$

The output spectrum $G_2(j\omega)$ is hence found by multiplying the magnitude characteristics of $G_1(j\omega)$ and $H(j\omega)$, and adding their phase characteristics. It is worth noting that the same output signal spectrum would be obtained if an input signal with spectrum $H(j\omega)$ was applied to a linear system described by the function $G_1(j\omega)$.

Hence a complex frequency function may describe a signal or a linear system. The complex function which defines the operation of a linear system as a function of frequency is called its " frequency response". The great attraction of the frequency-domain approach is that the spectrum of the output signal or waveform is equal to the product of that of the signal input and the system's frequency response. It is important to realize that no simple relationship of this sort exists in the time-domain, for example, it is not possible to derive the output waveform by multiplying the input wave form by some time function representing the system itself. The main disadvantage of the frequency-domain approach, however, is that we very often want to know how a particular waveform is modified as it passes through the system. In this case, we must first find its spectrum (that is , take its Fourier transform), then multiply by the system's frequency response to get the output spectrum, and finally take the inverse

2.2.2 Fourier transform to get the output time function

The method outlined above for describing the relationship between the input and the output signals of a linear system involve analysis in terms of continuous sinusoidal waveforms: the input and the output signals are represented as the sum of sets of continuous sinusoids (using the Fourier transform), and the linear system is represented by its effect on continuous sinusoids. This approach is often referred to as "steady-state" sinusoidal analysis, because the sinusoidal functions are assumed to exist throughout all time. A somewhat more general frequency-domain description of a linear system is obtained, using the Laplace transform. The Laplace transform gives a useful description of a signal waveform; not only does it allow us to derive a frequency-domain description of a certain signal for which the Fourier integral fails to converge, but it also gives us the possibility of representing a signal by a set of s-plane poles and zeros. If we have found the Laplace transform of a signal we may, as a general rule, derive its (sinusoidal) frequency spectrum by substitution of $j\omega$ for the complex frequency variable s: the Laplace transform may therefore be considered a more general frequency-domain description than the Fourier transform. A Laplace transform may also be used to describe either a signal or a system: when used to describe a system, it is generally referred to as a "transfer function". Just as the frequency response of a system defines its effects on continuous sinusoidal inputs, so the transfer function defines its effects on inputs of the form e^{st} (defined in the interval $0 < t < \infty$), where s may be real, imaginary or complex. Thus the transfer function defines the operation of a linear system for a rather wider range of input (and hence output) signal types than does the sinusoidal frequency response.

2.2.3 The time-domain approach

We are often interested in the modifications to a signal waveform caused by the system, and the transformation of time functions into their frequency domain equivalents is always possible but one can also work entirely with the time functions, even if the manipulation involved is by no means as simple as multiplication. In principle the time-domain approach is quite simple. we can consider an input signal to be composed of a succession of impulse functions, each of which generates a weighted version of the impulse response at the output of the linear system. The output signal is then found by the superposition of all such responses. This concept is, in a striking sense, the exact antithesis of the frequency-domain approach. For instead of considering the input to be made up of extended time functions (for example, continuous sinusoids), each of which has an infinitely narrow spectrum, we now think of it as being composed of extremely narrow time-pulses each of which has an infinitely broad spectrum. There are two common situations to which the time-domain approach is naturally suited; the first arises when a linear system is disturbed by an input of restricted duration, such as an isolated pulse or transient waveform; the second, when the input is in the form of a whole series of narrow pulses separated from one another in time, in other words, a sampled-data signal.

2.2.4 The processing of random signals

When a random waveform passes through a linear signal processor its time-averaged properties, as measured by its amplitude distribution, autocorrelation function or power spectrum are usually modified. It is a relatively straight forward matter to discuss the effect which a linear system has on the frequency and time-domain measures of a random signal. On the other hand, the effects of signal processing on such properties of a random waveform as its amplitude distribution or central moments are not generally easy to assess, because these properties are not simply related to its time and frequency domain structure. A random signal applied as input to a linear system, gives rize to modified but still random, output. As the power spectral density defines the average power of the various frequency components in a signal, but ignores their relative phases. Hence the power spectral density of the output from a linear system cannot depend upon the phase response of the system, but only upon its magnitude response. Suppose, for example, we apply a sinusoidal wave A in input of a linear system and obtain an output $A \cdot B \sin(\omega t + \varphi)$. The average input power $A^2/2$ and the average output power is $(AB)^2/2$.Therefore a system having a response magnitude B at a frequency ω modifies, the power of a component at that frequency by a factor B^2. More generally, if the input signal contains many frequency components and has a power spectral density $P_{xx}(\omega)$, then the output power spectral density will be

$$P_{yy}(\omega) = P_{xx}(\omega) \cdot |H(j\omega)|^2 = P_{xx}(\omega) H(j\omega) \cdot H^*(J\omega) \tag{4}$$

where $H(j\omega)$ is the frequency response of the system and $*$ denates the complex conjugate.

We have described how signals are modified by the system, there is another side of the same coin: knowledge of the properties of input and output signals allows up to infer those of an unknown linear system. For example, as mentioned earlier, we know that the input signal spectrum $G_1(j\omega)$ and the output signal spectrum $G_2(j\omega)$ of a system , therefore, we may express its response simply as

$$H(j\omega) = G_2(J\omega)/G_1(j\omega) \tag{5}$$

Similarly if a random signal with a power spectrum $P_{xx}(\omega)$ is applied to a linear system and we measure the output power spectrum $P_{yy}(\omega)$, we are in a position to define the system's response magnitude $|H(j\omega)|$ since

$$|H(j\omega)|^2 = \frac{P_{yy}(\omega)}{P_{xx}(\omega)} \tag{6}$$

Of course, any practical estimates of $P_{xx}(\omega)$ and $P_{yy}(\omega)$ based on finite portions of random input and output signals will contain sampling errors, so that the estimate of $|H(j\omega)|$ will only be approximately.

Unfortunately, comparison of input and output power spectra fails to reveal any information about the way in which a system modifies the phases of various frequency components applied to its input, and it is interesting to consider whether any other type of comparison of random input and output signals might be used to define the system's response in phase as well as in magnitude. Signal comparison using the cross spectral density and cross correlation functions show that these measures reflect not only magnitudes but also relative phases of common frequency components in the two signals $F_1(t)$ and $F_2(t)$ If we now consider $F_1(t)$ and $F_2(t)$ to represent random input and output wave forms on a linear system, it is clear that any measure which indicate relative phases of their various common components must indicate phase changes introduced by the system itself. This is the clue to a complete definition of a linear system by examination of the properties of its random input and output. Assume the time-domain system $f_1(t)$ output and the frequency-domain system $G_1(j\omega) - H(j\omega) - G2(j\omega)$ Recalling that the correlation and convolution are similar time-domain operations apart from reversal of one of the time functions, we may write down the following list of equivalent operations.

(a) cross-correlation of $f_1(t)$ and $f_2(t)$,

(b) convolution of $f_1(-t)$ and $f_2(t)$

(c) multiplication of $G_1(-j\omega)$ and $G_2(j\omega)$.

But $G_1(-j\omega) = G_1^*(j\omega)$ when $f_1(t)$ is a real time function: furthermore,

$$G_2(j\omega) = G_1(j\omega) \cdot H(j\omega) \tag{7}$$

so that

$$G_1(-j\omega) \cdot G_2(j\omega) = G_1^*(j\omega) \cdot G_1(j\omega) \cdot H(j\omega) = |G_1(j\omega)|^2 \cdot H(j\omega) \tag{8}$$

now $|G_1(j\omega)|^2$ is the power spectral density of the input signal, which is the Fourier transform of its auto correlation function $\gamma_{xx}(\tau)$: therefore operation (c) above is equivalent to

(d) multiplication of $|G_1(j\omega)|^2$ and $H(j\omega)$, and hence to:

(e) convolution of $\gamma(\tau)$ and $I(t)$.

This last result is most interesting: we started by assuming cross-correlation of the input and output wave forms of the system, and result (e) shows that this is equivalent to convoluting the impulse response of the linear system with the auto correlation function of its input. Formally the relationship may be stated by the convolution integral

$$\gamma_{xx}(\tau) = \int_{-\infty}^{\infty} \gamma_{xx}(\tau - T) \cdot I(T) \cdot dT \tag{9}$$

If we know the input auto-correlation function and evaluate the cross-correlation function relating the system input .$f_1(t)$ and its output $f_2(t)$ we may therefore evaluate the impulse response of the system. The above formula becomes particularly useful when the input $f_1(t)$ is a very wide band random signal, which has an auto-correlation function approximating a Dirac pulse at $\tau = 0$ In this case we have

$$\gamma_{xy}(\tau) = \int_{-\infty}^{\infty} \delta(\tau - T) \cdot I(T) \cdot dT \tag{10}$$

which, by the sifting property of the Dirac pulse, reduces directly to

$$\gamma_{xy}(\tau) = I(\tau) \tag{11}$$

In other words the input-output cross-correlation function has the same shape as the impulse response of the linear system. Since the cross-correlation function takes the same form as the system's impulse response only when the input random signal is wide band and in practice this means that the input power spectrum must be constant or "flat" over the full range of frequencies significantly transmitted by the system.

Although knowledge of the spectral properties of a linear system allows us to define its effect on the power spectrum (or auto correlation function) of a random signal, the same is unfortunately not true of the signal's amplitude distribution. The relationship between input and output signal amplitude distribution is not generally a simple one; neither is it unique. There is, however, one important type of amplitude distribution which is preserved by linear processing- namely the normal, or gaussian, distribution.

2.3 Non-linear systems

There is no general theoretical frame work available for the discussion of non-linear processes. A nonlinear process does not obey the principle of superposition, nor does it have the property of frequency - preservation: nonlinear systems may not be completely characterized by impulse or frequency response function, nor may their output signals be derived by transform methods or by convolution. However, it is sometimes useful to approximate a nonlinear input/output relationship by two or more linear characteristics, each of which applies to a certain range of input signal levels.

3 Overview of Calibrations

3.1 Requirements

The EFW instrument calibrations constitute a sub-set of the EFW instrument tests. The scope and procedures of the tests are based on the theoretical considerations described in section 2 and user requirements described in [Ref. **?**] and [Ref. **?**]. The list of requirements is found in [Ref. **?**] and [Ref. **?**].

3.2 Calibration Procedures and Products

The EFW instrument calibrations are based on the *digital tests* and the *analog tests.*

3.2.1 Digital calibarations

The major part of the instrument calibration comes from the *digital tests.* Test procedures as well as test products are described in [Ref. **?**].

3.2.2 Analog calibrations

The *analog tests* contribute teh phase response to the instrument calibrations. Test procedures as well as test products are described in [Ref. **?**].

3.3 Application of Calibrations

The use of the calibration material is described in [Ref. **?**].

4 Reference Documents