Tentamen för kursen Rymdfysik (1FA255)
2016-10-19

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
Anders Eriksson, David Andrews

Answers should be provided in Swedish or English.
Time: 08:00 - 13:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. This part is only graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Rockets are often launched eastward, to take advantage of the Earth's rotation.
   ii. The geostationary orbit is at about 300 km altitude.
   iii. The periapsis of a spacecraft in bound orbit is its closest point to the object it is orbiting.
   iv. The mass of a rocket (including fuel) scales exponentially with the velocity increase it provides.
   v. To raise the apogee of a satellite orbit in an economical way, one should fire a thruster (rocket) on a satellite when it is at the perigee, in a direction perpendicular to the direction of motion.

   (b) Consider a spacecraft in the shape of a sphere, of radius 1.238 m, with perfect internal heat conductivity. The spacecraft is located at 1 AU from the Sun but far from the Earth. Calculate the equilibrium temperature if $\alpha/\epsilon = 0.4$. The solar constant is 1370 W/m^2 and the Stefan-Boltzmann constant is 5.67×10^{-8} W m^{-2}K^{-4}.

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Plasmas are expected to be electrically neutral on length scales greater than the Debye length.
   ii. The Sun's magnetic dynamo is active mainly in the convective zone of the star.
   iii. At 1 AU the solar wind is dominated by its thermal energy density (rather than the magnetic energy density or bulk kinetic energy density)

iv. The temperature of the solar "atmosphere" reaches its maximum in the corona.

v. The "Parker spiral" pattern found in the heliosphere is caused by the Sun's magnetic field applying a force to the plasma, bending it into a spiral motion as seen from Earth.

(b) Saturn's largest moon Titan orbits the planet at a distance of 20 R_S. Usually, the moon is inside Saturn's magnetosphere (the radius of Saturn's dayside magnetopause $R_\mathrm{mp} \sim 22\ R_\mathrm{S}$), and the thick atmosphere of Titan is exposed to the energetic plasma found in Saturn's equatorial magnetosphere. Electrons in Saturn's equatorial magnetosphere may have thermal energies of $>10^3$ eV - would you expect them to be able to ionize neutral particles in Titan's atmosphere on impact? Name one other source of ionizing energy that Titan's atmosphere is exposed to.

Calculate the electromagnetic force on a newly-formed ion in Titan's atmosphere. Assume that Titan's orbital velocity is negligible relative to all relevant plasma flows.

Inside Saturn's magnetosphere at 20 R_S, in an inertial (non-rotating frame):

- $\mathbf{B} = (3, 3, 3)$ nT in spherical coordinates aligned with Saturn's spin axis (B_r, B_θ, B_ϕ).
- Saturn's magnetospheric plasma flow $\mathbf{v} = (0, 0, 100)$ km/s (v_r, v_θ, v_ϕ)

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. The Lorenz force only describes the force on a charged particle over timescales much longer than the gyroperiod.

ii. The "ExB" drift velocity is given by $\mathbf{v}_\mathrm{E} = \mathbf{E} \times \mathbf{B}/B^2$.

iii. The ∇B drift causes a current to flow in the plasma, because ions and electrons drift in opposite directions.

iv. The gyroradius of a particle increases as the magnetic field increases.

v. In general, the bounce period is longer than the drift period, for particles moving in dipolar fields.

vi. In the equatorial magnetosphere, the convection electric field causes plasma to drift towards the sun.

(b) Describe, in words and a drawing or two, the motion of positively charged ions and negatively charged electrons in a uniform, steady background magnetic field aligned with the Z direction, assuming that they each have zero kinetic energy parallel to this field. Draw a sketch of their motion. How does the motion change if the particles instead have a pitch angle $\alpha = 45°$?

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. An ionosphere will form around any planet with an atmosphere orbiting a star emitting ionizing radiation.

ii. The aurora is mainly created by ions accelerated downward in regions where Birkeland currents flow.

iii. In a magnetic substorm, the cross-tail current is disrupted and instead closes through field-aligned currents and the ionosphere.

iv. Dissociative recombination is an important plasma loss mechanism in the Earth's ionosphere.

v. The E layer in the Earth's ionosphere disappears quickly at night since the ionization source is no longer present and the recombination occurs fast at this altitude.

vi. Due to collisions, an electric field parallel to the magnetic field can drive a current in the ionosphere but not in the magnetosphere.

(b) Figure 1 shows the "current conditions" and "space weather forecast" from spaceweather.com on July 19, 2016. The forecast thus is for the day after, July 20. Figure 2 shows the solar wind and interplanetary magnetic field observed on ACE at L1 for July 19, 20, and 21. Compare the forecast to the ACE data. How does what actually happened compare to the forecast? What data or images presented on the spaceweather.com page was actually relevant for what happened? Do you think there was some geomagnetic storm and/or magnetospheric substorms in this period? When? Why? (Your answer should probably be somewhere between ten lines and one page)

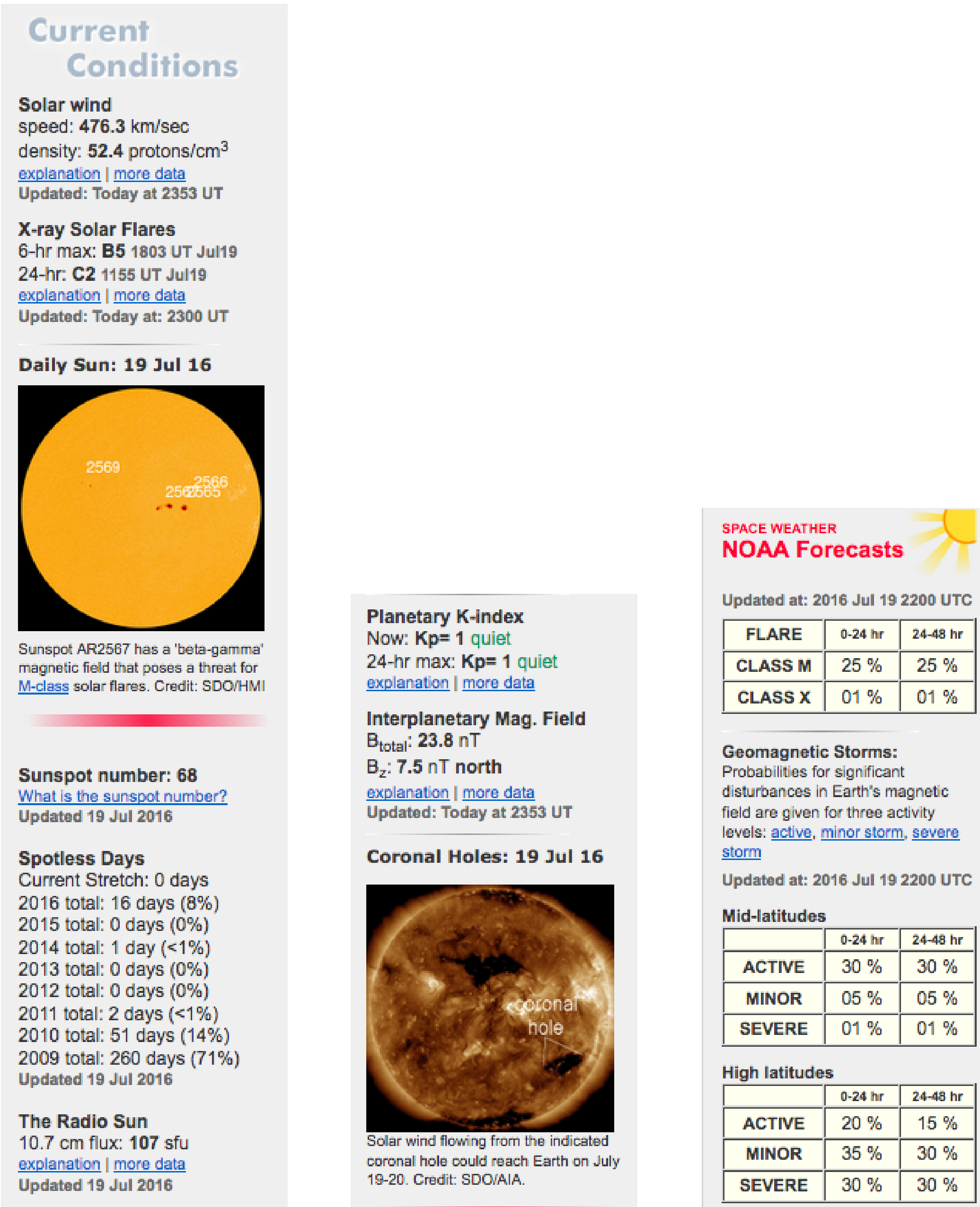

Figure 1: Current space weather conditions and forecast from July 19, 2016. [Source: http://www.spaceweather.com/]

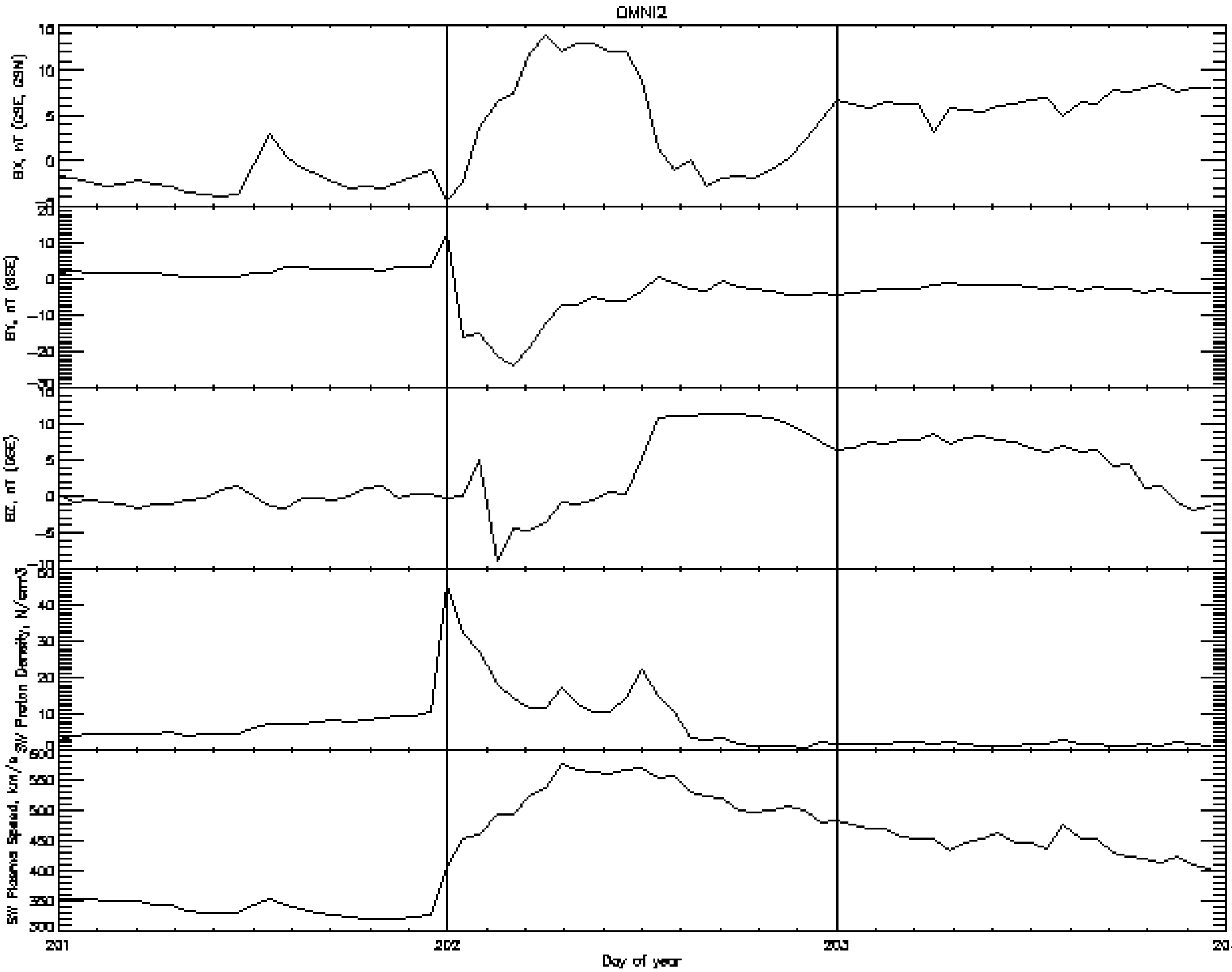

Figure 2: ACE data for July 19-21, 2016. [Source: OMNIWeb Plus, http://omniweb.gsfc.nasa.gov]

Part B

5. The geostationary satellite orbit is defined by the property that a spacecraft in this orbit stays above the same spot on the Earth all the time.

   (a) Calculate the radius of the geostationary orbit (in units of R_{E}. (1 p)

   (b) Assuming a dipolar geomagnetic field, at what latitude does a magnetic field line from geostationary orbit reach the ground? (2 p)

   (c) Why is a geostationary orbit necessarily equatorial? (1 p)

   (d) Do geostationary satellites ever enter eclipse (in the shadow of the Earth)? If so, does this happen to every geostationary satellite, and about how often do you expect it to occur? (2 p)

6. (a) Draw a sketch of the Earth's magnetosphere, viewed from the dusk side (so that the Sun is to the left of the page). Label the following regions / boundaries / features: Earth's magnetic field, magnetopause, bow shock, magnetosheath, magnetosphere, and stream lines of the solar wind. Show the direction of the major electric currents that flow *through* the page, and indicate the locations where magnetic reconnection can be important. (3 p)

   (b) By assuming a balance between the magnetic pressure of the Earth's magnetic field, and the dynamic (ram) pressure of the solar wind, calculate an expression for the distance to the magnetopause at the sub-solar point. How, if at all, does the magnetopause current layer affect this calculation? (3 p)

   (c) By considering the motion of charged particles from the solar wind encountering the magnetopause, show that the magnitude of the Chapman-Ferraro (magnetopause) current density is given by $j = 2mnv_{\perp}^2/B$, defining the terms in this expression. (2 p)

7. A spacecraft capable of detecting ions and electrons is located at $r = 5\ R_{\mathrm{E}}$ in the equatorial plane, at dawn (06:00 hours local time). Reconnection in the magnetotail causes high energy protons and electrons to be injected into the magnetosphere at midnight (00:00 hours local time), after which they begin to drift according to the ∇B force acting on them. The protons are injected with a range of energies from 1 to 10 keV, while the electrons have energies from 10 to 100 keV. Assume that all particles have a $90°$ pitch angle, and that the Earth's magnetic field is that of a dipole with equatorial field strength $B_{\mathrm{E}} = 31\ \mu$T, and Earth's radius $R_{\mathrm{E}} = 6370$ km.

   (a) Derive an expression for the time taken for these ions and electrons to drift once around the Earth. (2 p)

   (b) Which type of particles will the spacecraft detect first, and why? Will they be ions or electrons, and what energy will they have; 1, 10 or 100 keV? (1 p)

8. Consider the tail of the magnetosphere to consist of two lobes with oppositely directed magnetic field with strength 20 nT far away from the central plane. Assume that the current sheet between the lobes has a thickness of 5000 km in which the current is homogeneously distributed. Calculate the current density $\mathbf{j}$ and the variation of the magnetic field in this sheet. (3 p)

Lycka till!

.

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned}
\sigma_\mathrm{P} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \tfrac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\
\sigma_\mathrm{H} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \tfrac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\
\sigma_\parallel &= ne^2\left(\tfrac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \tfrac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right)
\end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}} I_{\mathrm{rad}}$$

aie090813