Tentamen för Rymdfysik I och NV1
2004-06-11

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 8:00 – 13:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilingual dictionary, for example English-Swedish or French-English, may also be used.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text (maybe more for the last question), possibly an equation or two and maybe a figure.

   (a) What is a magnetosphere? (1 p)

   (b) The escape velocity from the Earth's surface is some 11 km/s. Electrons and ions in the inner magnetosphere can have speeds much higher than so, but can still be trapped, not escaping out into interplanetary space. Why? (2 p)

   (c) Why are rockets often launched from places close to the equator? (2 p)

   (d) The solar wind temperature is typically some 100 000 K. How can a spacecraft survive in the solar wind? Why doesn't it just melt and evaporate? (2 p)

   (e) A coronal mass ejection occurs on the sun, at such a place that the increased solar wind is heading toward the Earth. What happens, and when? Describe typical space weather phenomena caused by this, including some effect on human systems. (3 p)

2. The Rosetta spacecraft was launched in March 2, 2004, on a mission to the comet 67P/Churyomov-Gerasimenko, which it will reach in the year 2014. For the moment, Rosetta is around 0.9 AU from the sun, which is as close as it will ever get, and hence the spacecraft is quite hot. Rosetta basically looks like a rectangular box, some $2 \times 2 \times 3$ m. It has been found that if one of the 2×3 m surfaces faces the sun, as was the original intention for the cruise phase, some items on this surface get too hot. To cool these items slightly, the spacecraft has been tilted. If Rosetta is turned $45\circ$ around its axis, so that two of the 2×3 m surfaces are sunlit with the sunlight hitting them at $45\mathrm{deg}$, what is the relative change in temperature of all the spacecraft? Does it increase or decrease? We assume that the spacecraft is well conducting, with all surfaces identical in material properties, and neglect any internal power dissipation. (4 p)

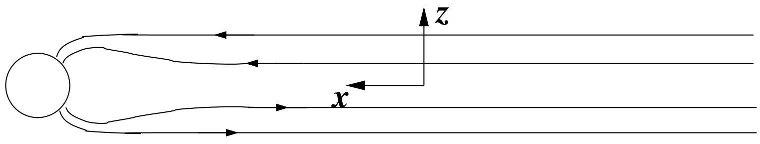

Figure 1: Idealized geometry of the relevant part of the geomagnetic tail.

3. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \le z \le a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

where $B_0 = 1$ nT, $a = 2000$ km and the coordinates are defined as in Figure 1.

(a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

(b) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. When the currents cannot flow through this region as before, where will they close now? Is this example relevant for any phenomenon in Earth's magnetosphere? (2 p)

4. A satellite orbits the Earth in a circular eastward equatorial orbit 1 R_E above the ground.

(a) How long is the orbit period (in hours)? (1 p)

(b) What do we call the plasma region through which the spacecraft is travelling? What velocity (relative to an inertial frame anchored in the Earth) do you expect the plasma to have here? Give a numerical value, with direction. (2 p)

(c) If the electric field in the rest frame of the plasma is zero, what is the electric field (magnitude and direction) observed by an instrument on the satellite? The geomagnetic field can be approximated with a dipole field with axis along Earth's rotation axis and strength 30 μT on the ground at the equator. (4 p)

5. At a position where the magnetic field is 2 % as strong as it is on the ground on the same field line, an electron initially having 20 eV kinetic energy and a pitch angle of 45° is accelerated downward along the magnetic field in a 1 kV potential drop which is concentrated to a very small region of space. Can the electron reach the ionosphere and contribute to the aurora? (4 p)

Lycka till!

Space Physics Formulas: Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_{\perp}^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force **F**:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F} \times \mathbf{B}}{q B^2}$$

Pitch angle:

$$\tan \alpha = v_{\perp} / v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 K T}{n e^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}} / (2\pi) = \frac{1}{2\pi} \sqrt{\frac{n e^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T \, \mathrm{d}t}{m_{\mathrm{fuel}} g} = v_{\mathrm{e}} / g$$

The rocket equation:

$$\Delta v = -g t_{\mathrm{burn}} + v_{\mathrm{e}} \ln \left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}} \right)$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon \sigma A_{\mathrm{e}} T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}} I_{\mathrm{rad}}$$