T 100315/1)

1. AB-
2. -B-
3. A-C
4. AB-
5. AB-
6. -B-
7. --C
8. ABC
9. AB-
10. --C

T 100315/2)

Conduction and convection can be neglected as means of cooling and heating the s/c in the tenuous space plasma, so only radiation remains. At equilibrium, the emitted power

$$P_e = A_e \varepsilon \sigma T^4 \tag{1}$$

must therefore equal the absorbed power

$$P_a = A_a \alpha I . \tag{2}$$

We know that

$$\begin{aligned} \alpha &= 0.5 \\ \varepsilon &= 0.2 . \end{aligned} \tag{3}$$

If the s/c side length is b, then

$$A_e = \text{total area of cube} = 6b^2 \tag{4}$$

while A_a (the absorbing area, i.e. the area projected to the sun) will vary from

$$A_a^{min} = b^2 \tag{5}$$

when one side faces the sun to

$$A_a^{max} = \sqrt{2}\, b^2 \tag{6}$$

when $\beta = 45°$. As we are at 2 AU, the intensity of sunlight is

$$I = \frac{I_0}{2^2} = \frac{1}{4} I_0 , \tag{7}$$

where $I_0 = 1370\ W/m^2$ is the intensity at 1 AU, becaus $I \propto 1/R^2$. Hence, $P_a = P_e$ gives

$$T = \left[\frac{A_a}{A_e} \cdot \frac{\alpha}{\varepsilon} \cdot \frac{I_0}{4\sigma}\right]^{1/4} \tag{8}$$

which gives

$$T^{max} = \left[\frac{A_a^{max}}{A_e} \cdot \frac{\alpha}{\varepsilon} \cdot \frac{I_0}{4\sigma}\right]^{1/4} =$$

$$= \left[\frac{\sqrt{2}}{6} \cdot \frac{0.5}{0.2} \cdot \frac{1370}{4 \cdot 5{,}67 \cdot 10^{-8}}\right] K \approx$$

$$\approx 244\ K \approx -29\ °C$$

and

$$T^{min} = \left[\frac{1}{\sqrt{2}}\right]^{1/4} \cdot T^{max} \approx 224\ K \approx$$

$$\approx -49\ °C$$

Answer: The temperature will vary between $-49°C$ for $\beta = 0 + n \cdot 90°$ and $-29°C$ for $\beta = 45° + n \cdot 90°$ $(n \in \mathbb{Z})$

T100315/3)

(a) From the formula sheet, we get the dipole field lines as

$$r = r_0 \sin^2\theta \qquad (1)$$

where r_0 is the field line geocentric distance at $\theta = 90^\circ$, i.e. in the equatorial plane. Our field line has $\theta = 90^\circ - 60^\circ = 30^\circ$ at $r = R_E$, so we get

$$r_0 = \frac{r}{\sin^2\theta} = \frac{R_E}{\sin^2 30^\circ} = \underline{\underline{4 R_E.}} \qquad (2)$$

The pitch angle evolution is governed by the combined conservation laws for orbital magnetic moment,

$$\mu = \frac{\frac{1}{2} m v_\perp^2}{B} = \frac{\frac{1}{2} m v^2 \sin^2\alpha}{B} = \text{const} \qquad (3)$$

and kinetic energy

$$K = \frac{1}{2} m v^2 = \text{const}, \qquad (4)$$

resulting in

$$\frac{\sin^2\alpha}{B} = \text{const}. \qquad (5)$$

Hence, as $\alpha = 90^\circ$ at the mirror point, its value at the equatorial plane is given by

$$\sin^2\alpha_{eq} = \frac{B_{eq}}{B_m} \qquad (6)$$

where

$$B_{eq} = B_0\left(\frac{R_E}{r}\right)^3 = B_0\left(\frac{R_E}{4R_E}\right)^3 = \frac{B_0}{64} \tag{7}$$

is the B-field strength where the field line reaches the equatorial plane. The corresponding quantity at the mirror point, B_m, can be found if we evaluate B as a function of altitude along this field line. We have

$$\bar{B} = -B_0\left(\frac{R_E}{r}\right)^3\left[2\hat{r}\cos\theta + \hat{\theta}\sin\theta\right] \tag{8}$$

$$B = B_0\left(\frac{R_E}{r}\right)^3\left[4\cos^2\theta + \sin^2\theta\right]^{1/2} =$$

$$= B_0\left(\frac{R_E}{r}\right)^3\left[4 - 3\sin^2\theta\right]^{1/2} =$$

$$= B_0\left(\frac{R_E}{r}\right)^3\left[4 - 3\frac{r}{r_0}\right]^{1/2} \tag{9}$$

where we in the last step used equation (1). With (2),

$$B = B_0\left(\frac{R_E}{r}\right)^3\left[4 - \frac{3}{4}\frac{r}{R_E}\right]^{1/2}, \tag{10}$$

and we get from (6) the desired pitch angle in the equatorial plane as

$$\sin^2\alpha_{eq} = \frac{B_{eq}}{B_m} = \frac{1}{64}\left(\frac{r_m}{R_E}\right)^3\frac{1}{\sqrt{4 - \frac{3r_m}{4R_E}}} \tag{11}$$

where the mirror point radial distance is given by

$$r_m = R_E + h = (6371.2 + 10000)\ \text{km}. \tag{12}$$

Thus,

$$\sin^2\alpha_{eq} = \frac{1}{64}\left(\frac{6371.2 + 10\,000}{6371.2}\right)^3\frac{1}{\sqrt{4 - \frac{3\cdot(6371.2 + 10000)}{4\cdot 6371.2}}}$$

$$\underline{\underline{\alpha_{eq} \approx 25^\circ}}$$

(b) We can once again use (5), giving

$$\frac{\sin^2 60^\circ}{B_m^{old}} = \frac{1}{B_m^{new}}, \tag{13}$$

as $\alpha = 90^\circ$ at the new mirror point. We thus get

$$B_m^{new} = \frac{B_m^{old}}{\sin^2 60^\circ} = \frac{4}{3} B_m^{old}. \tag{14}$$

Using (10), we have

$$\frac{B_m^{new}}{B_0} = \left(\frac{R_E}{r}\right)^3 \left[4 - \frac{3}{4}\frac{r}{R_E}\right]^{1/2} \tag{15}.$$

While there is no analytical solution to this equation, it can conveniently be solved by iteration. Putting

$$b = \frac{B_m^{new}}{B_0} = \frac{4}{3}\frac{B_m^{old}}{B_0} = \frac{4}{3}\left(\frac{R_E}{R_E+h}\right)^3 \sqrt{4 - \frac{3}{4}\frac{R_E+h}{R_E}} =$$

$$= \frac{4}{3}\left(\frac{6371.2}{16371.2}\right)^3 \sqrt{4 - \frac{3}{4}\cdot\frac{16371.2}{6371.2}} = 0.1131 \tag{16}$$

and

$$x = r/R_E, \tag{17}$$

(15) can be recast as

$$x^3 = \frac{\sqrt{4 - \frac{3}{4}x}}{b} \tag{18}$$

$$x = b^{-1/3}\left(4 - \frac{3}{4}x\right)^{1/6} \tag{19}$$

Iterating this as

$$x_{n+1} = b^{-1/3}\left(4 - \frac{3}{4}x_n\right)^{1/6} \tag{20}$$

with $x_0 = 0$ gives

$x_0 = 0$

$x_1 = 2.6049$

$x_2 = 2.3296$

$x_3 = 2.3672$

$x_4 = 2.3623$

$x_5 = 2.3629$

$x_6 = 2.3628$

$x_7 = 2.3628$

$x_8 = 2.3628$

$x_9 = 2.3628$

We thus get the altitude of the new mirror point as

$x \approx 2.3628$

$r \approx 2.3628 \ R_E$

$$h \approx (2.3628 - 1) \ R_E = 1.3628 \cdot 6371.2 \text{ km} \approx \underline{\underline{8683 \text{ km}}}$$

T 100315/4)

(a) As the orbit is circular, the spacecraft has a constant acceleration

$$a = \frac{v^2}{r} \qquad (1)$$

where the velocity v can be expressed in terms of the orbital radius r and period T as

$$v = \frac{2\pi r}{T} \qquad (2)$$

so that

$$a = \frac{4\pi^2 r}{T^2}. \qquad (3)$$

This acceleration must be provided by the gravitational force from the Earth, i.e.

$$a = \frac{F}{m} = \frac{GM}{r^2} \qquad (4)$$

where $M = 6 \cdot 10^{24}$ kg is the mass of the Earth. Hence,

$$T = 2\pi\sqrt{\frac{r}{a}} = 2\pi\sqrt{\frac{r^3}{GM}} \qquad (5)$$

$$T_{upper} = 2\pi\sqrt{\frac{(6371.2+550)^3 \cdot 10^9}{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}}\ s \approx 5719\ s$$

$$T_{lower} = 2\pi\sqrt{\frac{(6371.2+450)^3 \cdot 10^9}{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}}\ s \approx 5595\ s$$

$$T_{upper} - T_{lower} \approx 5719 - 5595\ s = 124\ s \approx \underline{\underline{2\ min}}$$

(b) In a dipole field,

$$\bar{B} = -B_0\left(\frac{R_E}{r}\right)^3 (2\hat{r}\cos\theta + \hat{\theta}\sin\theta) \qquad (6)$$

and

$$B = |\bar{B}| = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{4\cos^2\theta + \sin^2\theta} =$$

$$= B_0\left(\frac{R_E}{r}\right)\sqrt{1+3\cos^2\theta} \qquad (7)$$

For any fixed r, B is maximum when $\cos^2\theta$ is, i.e. at $\theta = 0^\circ$ and $\theta = 180^\circ$ which is over the poles (latitude $\pm 90^\circ$). The field strength here is

$$B = B_0\left(\frac{R_E}{r}\right)^3 \cdot 2 =$$

$$= 2 \cdot 30 \left(\frac{6371.2}{6371.2+450}\right)^3 \mu T \approx 49\ \mu T$$

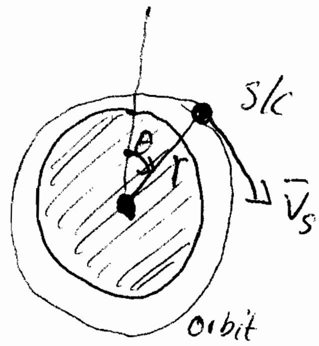
(c) Assuming that the electric field in the plasma frame of reference is zero, the field seen by the satellite satisfies

$$\bar{E} + (\bar{v}_p - \bar{v}_s) \times \bar{B} = 0 \qquad (8)$$

where $\bar{v}_p - \bar{v}_s$ is the velocity of the plasma as seen in the s/c frame. With the s/c velocity $\bar{v}_s$ given by

$$\bar{v}_s = \pm v\hat{\theta} \qquad (9)$$

because of the polar orbit, where v is given by (2), and the plasma velocity by



$$\bar{v}_p = \frac{2\pi r \sin\theta}{T_E}\hat{\varphi} = v_p\hat{\varphi} \qquad (10)$$

because the circle the plasma follows when corotating with the Earth has radius $r\sin\theta$ (T_E = Earth rotation period), we get

$$\bar{E} = (\bar{v}_s - \bar{v}_p)\times\bar{B}. \qquad (11)$$

Using the dipole field (6), we get

$$\bar{E} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{\varphi} \\ 0 & \pm v & -v_p \\ B_r & B_\theta & 0 \end{vmatrix} =$$

$$= v_p B_\theta \hat{r} - v_p B_r \hat{\theta} \mp v B_r \hat{\varphi} \qquad (12)$$

The horizontal component of $\bar{E}$ thus has strength

$$E_{hor} = \sqrt{E_\theta^2 + E_\varphi^2} = |B_r|\sqrt{v_p^2 + v^2} =$$

$$= 2B_0\left(\frac{R_E}{r}\right)^3 |\cos\theta| \sqrt{v_0^2\sin^2\theta + v^2} \qquad (13)$$

where

$$v_0 = \frac{2\pi r}{T_e}. \qquad (14)$$

Now the numerical values are

$$V_0 = \frac{2\pi r}{T_E} = \frac{2\pi(6371.2+450)}{24\cdot 3600}\ km/s \approx 0.50\ km/s$$

$$V = \frac{2\pi r}{T} = \sqrt{\frac{GM}{r}} = \sqrt{\frac{6.67\cdot 10^{-11}\cdot 6\cdot 10^{24}}{(6371.2+450)\cdot 10^{3}}}\ m/s \approx 7.7\ km/s$$

which means that the $V_0^2 \sin^2\theta$ term inside the square root in (13) is a small correction. We can therefore suspect that the maxima of (13) will be where $|\cos\theta|$ maximizes (the minimum clearly is where $|\cos\theta|$ minimizes, i.e. at the equator, where the s/c moves parallell to the B-field lines and hence sees no electric field).

Putting

$$y = E_{hor}^2 / \left[4B_0^2 \left(\frac{R_E}{r}\right)^6\right] \qquad (15)$$

and

$$x = \cos^2\theta \qquad (16)$$

we get from (13) that

$$y = \cos^2\theta \left(V_0^2 \sin^2\theta + V^2\right) = \cos^2\theta \left(V_0^2 + V^2 - V_0^2\cos^2\theta\right) =$$

$$= x\left(V^2 + V_0^2 - V_0^2 x\right) = V^2 x\left(1 + \frac{V_0^2}{V^2} - \frac{V_0^2}{V^2}x\right) \qquad (17)$$

with extrema at

$$0 = \frac{dy}{dx} = V^2\left[1 + \frac{V_0^2}{V^2} - \frac{V_0^2}{V^2}x - \frac{V_0^2}{V^2}x\right] = V^2\left[1 + \frac{V_0^2}{V^2} - 2\frac{V_0^2}{V^2}x\right] \qquad (18)$$

$$\Rightarrow x = \frac{V_0^2 + V^2}{2V_0^2} > 1, \qquad (19)$$

but as $x = \cos^2\theta$, this shows that there indeed are no maxima inside the intervals $]0, 90°[$ or $]90°, 180°[$. The maxima thus are at the poles (latitude $\pm 90°$) with strength

$$E_{hor}^{max} = 2B_0\left(\frac{R_E}{r}\right)^3 V = 2\cdot 30\cdot 10^{-6}\left(\frac{6371.2}{6371.2+450}\right)^3 \cdot 7.7\cdot 10^3\ V/m \approx \underline{\underline{0.38\ V/m}}$$

T 100315/5)

(a) We estimate this distance by assuming a pressure balance between the dynamic pressure of the incoming solar wind,

$$p_d^{sw} \approx \frac{1}{2} m n v^2 \qquad (1)$$

where m is taken to be the proton mass and we get

$n = 1.2 \text{ cm}^{-3} = 1.2 \cdot 10^6 \text{ m}^{-3}$

$v = 424 \text{ km/s}$

from the data, and the magnetic pressure inside the magnetosphere, estimated as

$$p_B^M \approx \frac{B_0^2}{2\mu_0} \left(\frac{R_E}{r}\right)^6. \qquad (2)$$

As p_B^M depends on r while p_d^{sw} does not, we get

$$r \approx R_E \left(\frac{B_0^2}{\mu_0 m n v^2}\right)^{1/6} \approx$$

$$\approx \left(\frac{(30 \cdot 10^{-6})^2}{4\pi \cdot 10^{-7} \cdot 1{,}67 \cdot 10^{-27} \cdot 1.2 \cdot 10^6 \cdot (424 \cdot 10^3)^2}\right)^{1/6} R_E$$

$$\approx \underline{\underline{11.2 \; R_E}}$$

(b) Solar flare probability: Rapid growth of the sunspot 1054 may lead to a solar flare eruption. Statistically, solar flares are linked to sunspots and to the evolution of sunspots.

Geomagnetic disturbance forecast: Geomagnetic disturbances come in two classes:

(i) Solar-induced geomagnetic storms, caused by some solar wind structure hitting the magnetosphere. Common causes are CMEs (coronal mass ejections), flares and fast solar wind streams from coronal holes

(ii) Geomagnetic substorms, which constitute an internal dynamical mode of the magnetosphere, but which are regulated by the z-component of the interplanetary magnetic field (IMF B_z).

In this case, IMF $B_z > 0$ (northward), meaning that little loading of the geomagnetic tail with magnetic flux takes place, and there is no imminent risk for substorms. No large coronal holes face the Earth, but the possibility of a flare of significant size means there is an increased risk for geomagnetic storms on the timescale of 24-48 hours, as disturbances from the sun takes about that time to reach Earth.